USESO 2021

Astronomy
KEY

Instructions:

- Section I consists of 10 multiple choice questions with each question worth 2 points. There is only one correct option on multiple choice questions.
- Section II consists of 2 multipart, free-response questions.
- A non-graphing, non-programmable calculator is allowed; show all work for calculations.
- Recommended time management: 30 minutes on each section.

Section I

1. (2 points) A Northern Hemisphere observer records a low tide at solar noon. Assuming a semidiurnal tidal pattern, what could the observer see in the sky six hours later? (*Note*: Assume equilibrium tides)

Photo A

Photo B

I) The moon in Photo A high in the sky

II) The moon in Photo B high in the sky

III) No moon visible

A. I only

B. II only

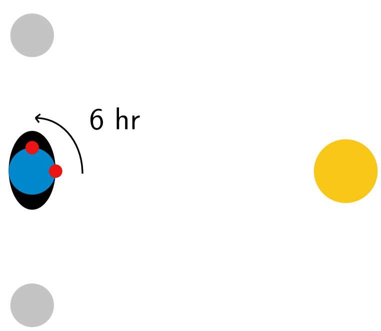
C. III only

D. I or II

E. I or III

F. II or III

Solution:



Because there is low tide at solar noon, the moon must either be at first or third quarter (assuming equilibrium tides). In six hours, the observer will have rotated into one of the tidal bulges (high tide). Photo A shows a first quarter moon in the Northern Hemisphere, which would be visible. However, a third quarter moon would only be visible on the opposite side of Earth. Because the moon can either be at first or third quarter, the answer is $\boxed{\text{E}}$.

2. (2 points) You travel to a parallel universe. Through your astronomical wisdom, you realize that the alternate Earth experiences retrograde motion, the eccentricity of the its orbit is decreasing from 0.01 to 0.002, its axial tilt is increasing from 20.5° to 29°, and, incredibly, it lacks axial precession. Assuming all other things equal, how might global climate change on the alternate Earth?

   A. Milder seasons and overall warming trend

   B. More extreme seasons and overall warming trend

   C. Milder seasons and overall cooling trend

   D. More extreme seasons and overall cooling trend

> **Solution:** Increasing the axial tilt will lead to a greater interhemispheric insolation difference near the solstices. Thus, there will be more extreme seasons. Decreasing the eccentricity will lead to a slight warming because the Earth will spend more time closer to the Sun.

3. (2 points) Mars is thought to be currently dominated by cold-based glaciers, glaciers that exist in subfreezing temperatures and have minimal meltwater. From this information, which of the following is most reasonable to infer?

   A. Compared to glaciers on Earth, internal deformation within these glaciers is very low.

   B. There is little alteration of the topography underneath the glaciers.

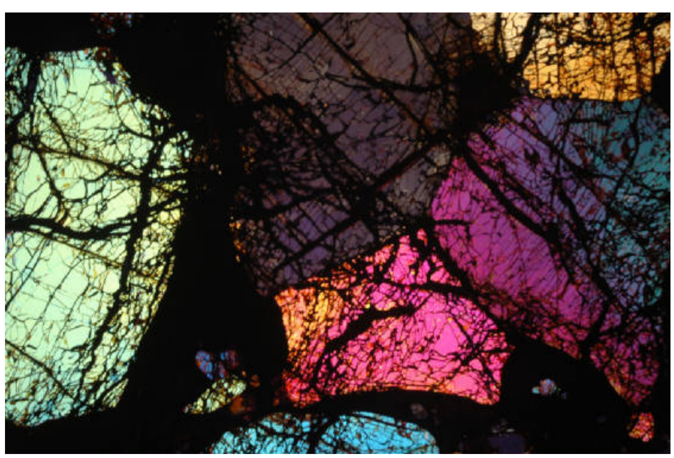
   C. Glaciers on Mars are mostly static as they do not easily flow.

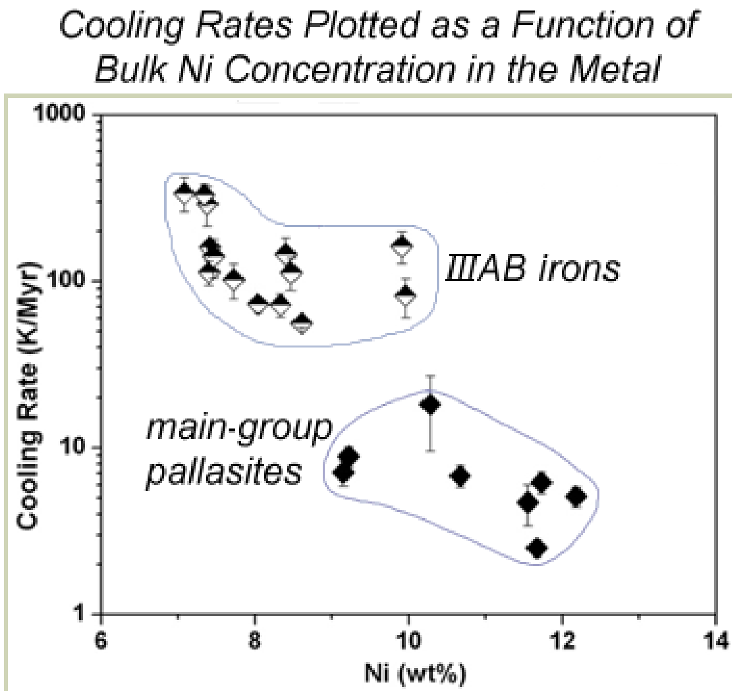
   D. Eskers are actively forming depositional features.

   E. If the glaciers were to sublimate and retreat, the most prominent glacial features would be erosional.

> **Solution:** Cold-based glaciers are largely frozen to the ground and flow by plastic deformation. With little meltwater to aid in basal slip and erode their terrain, the underlying topography is thought to be preserved well.

4. (2 points) The figure below is a thin section of the Huckitta pallasite, a main group pallasite meteorite. The colored mineral is forsterite (Mg-rich olivine), while the black mineral is an iron-nickel alloy.





(Adapted from Yang *et al*., 2010, *GCA*, doi:10.1016/j.gca.2010.04.016.)

IIIAB irons are a population of iron meteorites, which are composed of an iron-nickel alloy of kamacite and taenite. IIIAB irons and main-group pallasites are plotted above in terms of their cooling rate and nickel content. Which of the following can be reasonably inferred?

I) Main-group pallasites originate from the core-mantle boundary of a differentiated parent body

II) IIIAB irons and main-group pallasites originated from the same parent body

A. I only

B. II only

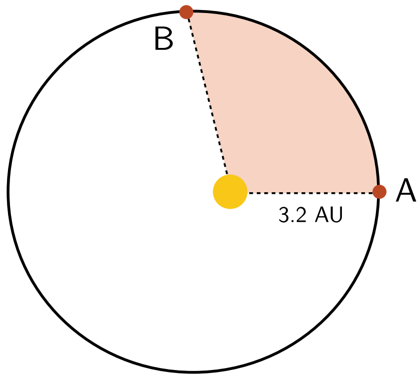
C. I and II

D. Neither I nor II

Solution: As seen in the thin section, pallasites consist of olivine (typical of planetary mantles) and iron-nickel alloy (typical of planetary cores). I is one way by which pallasites may form.

As seen in the second figure, IIIAB irons cooled considerably faster (an order of magnitude) than main-group pallasites. If they originated from the same parent body, IIIAB irons (formed from the core) would need to be interior to pallasites (formed from the CMB), and we would expect the IIIAB irons to have cooled slower than main-group pallasites. This is inconsistent with the inferred cooling rates, so II is likely false.

5. (2 points) A planet P orbits the Sun with a semi-major axis of 4 astronomical units (AU). At time $t = 0$, planet P is at perihelion (location "A"), 3.2 AU from the sun. At $t = 1.5$ years, planet P is at location B in its orbit.



What is the area between the following boundaries: B-Sun, A-Sun, and the orbital path from A to B (shown in figure above). Give your answer to the nearest tenth in terms of π. (*Note*: The area of an ellipse is given by $A = \pi ab$ where a and b are the semimajor and semiminor axes, respectively)

A. 2.4π AU^2

B. 2.8π AU^2

C. 2.9π AU^2

D. 3.6π AU^2

E. 3.8π AU^2

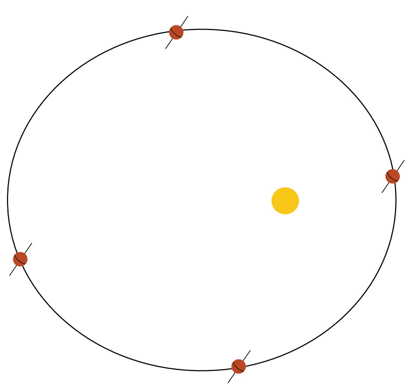
F. 4.0π AU^2

Solution: By Kepler's Second Law, the ratio of the sector area to the total ellipse area is equal to the ratio of the time elapsed from A to B to the total orbital period. As such, $A = \frac{\pi ab \cdot 1.5}{T}$, where A is the area of the highlighted sector. By Kepler's Third Law, $T = 4^{3/2} = 8$ years.

We know the semimajor axis a is 4 AU. Further, $c^2 = a^2 - b^2$, where c is the distance from the center to one focus (this can be deduced from the fact that the sum of the distances from the ellipse to the foci is constant). Thus, $b = \sqrt{a^2 - c^2} = 3.92$ AU.

Finally, $A = \frac{\pi(4)(3.92) \cdot 1.5}{8} = \boxed{2.9\pi \text{ AU}^2}$.

6. (2 points) Close to the solstices, Mars's atmospheric circulation is characterized by a large cross-equatorial Hadley cell. Its orbit (eccentricity exaggerated), as well as its rotational axis during the equinoxes and solstices, is shown below.



Which of the following is **not** true about Mars's seasons and atmospheric dynamics?

A. Southern summer is warmer than northern summer.

B. Southern winter is longer than northern winter.

C. During the equinoxes, there are two Hadley cells with rising air near the equator.

D. Hadley circulation during southern summer is stronger than northern summer because the southern hemisphere is significantly elevated relative to the northern hemisphere.

E. The northern ice cap is significantly larger than the southern ice cap due to net northward water vapor advection by climatological mean atmospheric circulation.

F. None of the above.

Solution: A is true because Mars is closer to the Sun during southern summer. Likewise, B is true because Mars is further from the Sun during southern winter than northern winter. C is reasonable because the zone of maximum insolation occurs near the equator when Mars is near its equinoxes. D is tricky - Mars does indeed have a significantly elevated southern hemisphere (by a few km; see Mars hemispheric dichotomy), but the effect of topography on atmospheric circulation may be complicated. The elevated SH makes it easier for air to rise during southern summer and blocks southward flowing air during northern summer. E is true - because southern summer Hadley circulation is stronger, water vapor and carbon dioxide are advected northward and deposited on the Northern ice cap. Hence, the answer is $\boxed{\text{F}}$.

7. (2 points) Rachmaninoff Crater (**Figure 1a**) is a peak-ring crater on Mercury, hypothesized to show recent (1.0 Ga) volcanic activity. Concentric graben and ridge structures have also been observed within the peak-ring (**Figure 1b**).

Though not yet confirmed, assume that volcanism associated with Rachmaninoff Crater originated from within the peak-ring structure (at B) and flowed out into the space between the peak-ring and the outer crater rim (at A).

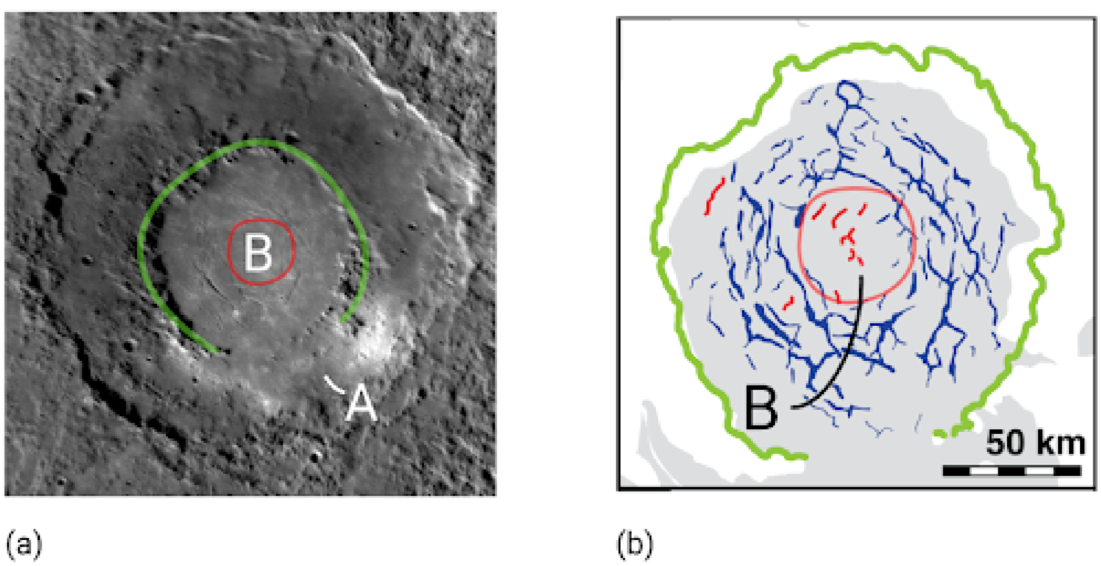


Figure 1 [modified after Blair et al.]: (a) Shows a mosaic of the Rachmaninoff impact basin. Note the peak-ring structure, highlighted in green, and a cooled lava flow denoted with letter A. The green and red outlines correspond to (b). (b) Shows the graben (blue) and wrinkle ridge (red) structures within the peak-ring (green).

Magmas of Rachmaninoff Crater are low in Fe and can be approximately characterized by a CMS (CaO-MgO-SiO_2) three-component system. Which of the following is true?

I) Thermal contraction of cooling lava arising from B generates large extensional stresses in the rest of the peak-ring.

II) Rachmaninoff Crater is undergoing rapid crustal subsidence.

III) The composition of rocks at A is characterized by a higher CaO + MgO to SiO_2 ratio than rocks at B.

A. I only

B. II only

C. III only

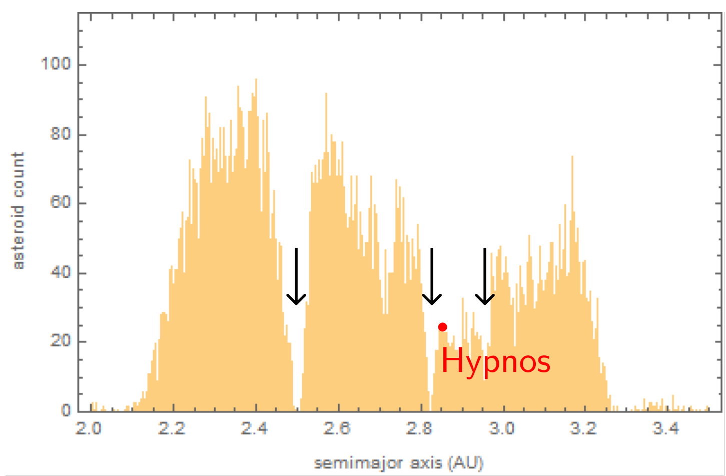
D. I and II

E. I and III

F. II and III

Solution: As shown in (b), there are contractional wrinkle ridge features in B. B is surrounded by graben, which are extensional structures. I provides a reasonable explanation for these two observations. II is not true because craters undergo isostatic rebound following formation. III is also not true: silica-poor minerals crystallize out of melt first, and lava at A flowed from B, so the composition of rocks at A will have a lower CaO + MgO to SiO_2 ratio than rocks at B.

8. (2 points) The figure below shows the distribution of main-belt asteroids.



(a) Which of the following accounts for the gaps in the distribution, as marked by the arrows?

A. Mean motion resonance with Mars

B. Mean motion resonance with Jupiter

C. Shepherding (i.e., orbit clearing) by large asteroids like Ceres and Pallas

D. Radiation pressure from solar wind

Solution: Orbital resonances with Jupiter render orbits with particular periods—related to the length of semimajor axes by Kepler's 3rd law—unstable, creating gaps in the asteroid belt named Kirkwood gaps.

(b) The asteroid Hypnos has been proposed to be an "extinct" or possibly dormant comet whose surface volatiles have at least mostly sublimated. Which of the following about Hypnos supports its status as a comet nucleus?

A. Diameter: 520 meters

B. Albedo: 0.057

C. Inclination: 1.981°

D. Perihelion/aphelion: 0.949/4.732 AU

E. Orbital period: 4.97 years

Solution: Most asteroids have low eccentricities. However, Hypnos has a very high eccentricity typical of comets, as seen in the large difference between its perihelion and aphelion.

9. (2 points) On June 3rd, 2020, the phase of the Earth is full as observed from Venus, and the phase of Venus is new as observed from Earth. When is next time Earth will be in its full phase as observed from Venus?

Earth orbital period	365.25 days
Venus orbital period	224.65 days

A. October 5, 2020

B. January 13, 2021

C. March 22, 2021

D. August 25, 2021

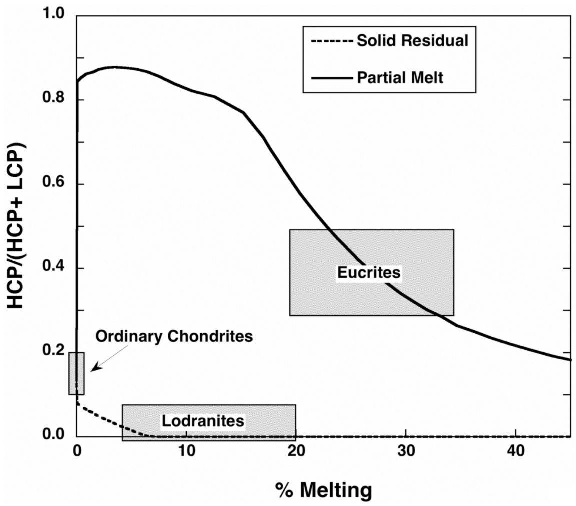
E. January 7, 2022

Solution: Earth is full as seen from Venus when Earth, Venus, and Sun, are in a line. This can occur at either conjunction or opposition. Because Earth is full and Venus is new on June 3rd, Venus must be between the Sun and Earth (Earth is in opposition relative to Venus). To find the date of the next conjunction, we need to consider the revolution of both Earth and Venus. We can find the angle between Venus, the Sun, and Earth by

$$\frac{\theta}{t} = \frac{2\pi}{224.65} - \frac{2\pi}{365.25} \tag{1}$$

The next conjunction occurs when $\theta = \pi$. Plugging in θ we find that $t = 291.8$ days. This corresponds to March 22, 2021.

10. (2 points) High-calcium pyroxene (HCP) and low-calcium pyroxene (LCP) are spectrally detectable, making them useful indicators in extraterrestrial igneous studies. Spectroscopic pyroxene analysis has been applied in studies of both S-type asteroids (**Figure 1**) and Martian paleovolcanism (**Figure 2**).



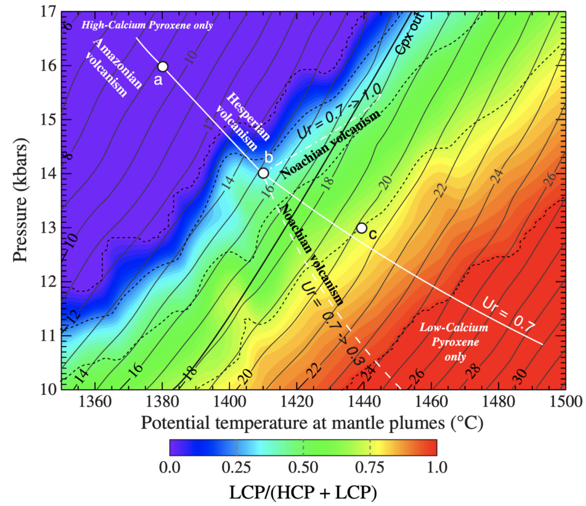


Figure 1, left: Calculations based on two achondritic meteorite groups, eucrites and lodranites, used to determine the relationship between the extent of partial melting and the ratio of high-calcium pyroxene to total pyroxene. **Figure 2**, right: Pyroxene composition, represented by the ratio of low-calcium pyroxene to total pyroxene, corresponding to volcanic deposits from the Noachian, Hesperian, and Amazonian Periods of the Martian Geologic Time Scale.

(a) Based on the general relationship shown in **Figure 1**, which of the following is true?

A. Noachian volcanic deposits exhibit a higher degree of partial melting than Hesperian deposits

B. Hesperian volcanic deposits exhibit a higher degree of partial melting than Noachian deposits

C. Both Noachian and Hesperian volcanic deposits exhibit the same degree of partial melting

D. Neither Noachian nor Hesperian volcanic deposits exhibit partial melting

Solution: Figure 1 shows that lower HCP/(HCP+LCP) values generally correspond with greater melting percentage. Noachian volcanic deposits have lower HCP/(HCP+LCP) than Hesperian deposits. Thus, the answer is $\boxed{\text{A}}$.

(b) Which of the following is a plausible explanation for the change in pyroxene composition between the Noachian and Hesperian periods?

A. Shock heating from supernovae occurred during the Hesperian Period

B. Tidal heating due to interactions with Phobos during was strong during the Hesperian Period

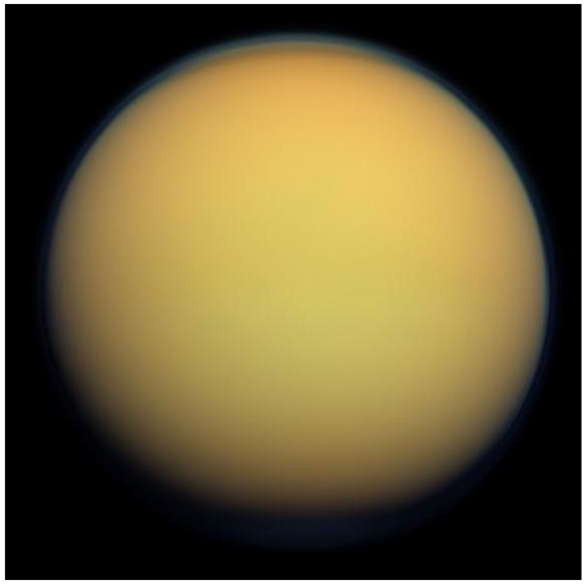
C. Mantle pressures were greater during the Noachian Period

D. Mantle temperatures were greater during the Noachian Period

Solution: A and B are very unreasonable (supernovae are transient phenomena and do not affect partial melting, and Phobos is far too small to have any significant tidal heating effect on Mars). C is also unreasonable, as mantle pressures do not change significantly over time. Hence $\boxed{\text{D}}$ is the only reasonable option; indeed, the Noachian is the earliest Martian period, so greater mantle temperatures than the Hesperian would be expected.

Section II: Problem 1

Question	1	2	3	4	5	6	7	Total
Points	3	2	3	2	2	1	2	15 (30%)



Titan, shown above, is a moon of Saturn. It has a thick atmosphere and its surface is modified by (hydrocarbon) hydrologic processes akin to those on Earth.

1. (3 points) Using the orbital parameters given below, calculate the orbital period, to the nearest day, of Titan.
   - Semi-major axis of Titan $= 1.22 \cdot 10^9$ m
   - The gravitational constant $G = 6.67 \cdot 10^{-11}$ N m/kg^2
   - Mass of Saturn $= 5.683 \cdot 10^{26}$ kg
   - Mass of Titan $= 1.345 \cdot 10^{23}$ kg

   Note: if you could not complete question 1, use 15 days as the orbital period of Titan for the following questions.

   Solution: By Kepler's Third Law, $T^2 = \frac{4\pi^2}{G(M+m)}a^3 \approx \frac{4\pi^2}{GM}a^3$ where M is the mass of Saturn. Plugging in numbers, we find that $T = 1.375 \cdot 10^6$ seconds. This is approximately $\boxed{16}$ days.

Titan is tidally locked and rotates synchronously with Saturn (similar to the Earth's moon), but it still exhibits considerable tides due to its eccentricity ($e = 0.0288$). This eccentricity tide can be broken into two components: (1) the radial tide, due to variations in distance from Saturn; (2) the librational tide, which we'll explore later.

2. (2 points) What is the period of the radial tide? Briefly explain.

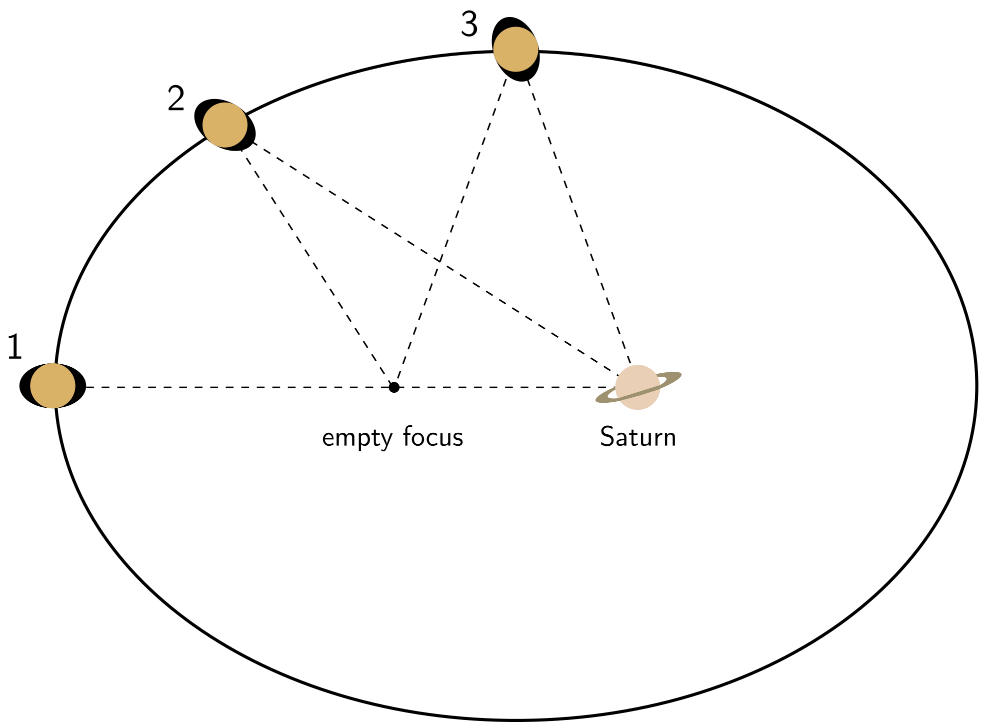
   Solution: The maximum radial tide occurs at periapsis, and the minimum radial tide occurs at apoapsis. Thus, the period of the radial tide is equal to the period of revolution, or $\boxed{16}$ days. (15 days if (1) is incomplete)

3. (3 points) How much greater is the maximum tidal force than the minimum tidal force of Titan's radial tide? Give your answer as a percentage. (*Hint:* $\frac{r_a}{r_p} = \frac{1-e}{1+e}$)

Solution: Tidal force is proportional to the inverse cube of the distance between the two bodies. Hence, $\frac{T_a}{T_p} = \left(\frac{1+e}{1-e}\right)^3 = 1.189$. $1.189 - 1 = 0.189 \longrightarrow \boxed{18.9\%}$. (Note that this is sometimes approximated as $6e$; indeed, for small x, a first order expansion of $\left(\frac{1+x}{1-x}\right)^n$ is $1 + 2nx$)

4. (2 points) The librational tide arises from the interesting fact that Titan rotates such that it always faces the empty focus, not Saturn itself. Hence, the tidal bulge oscillates (librates) along Titan's equator. What is the period of the librational tide? Briefly explain.

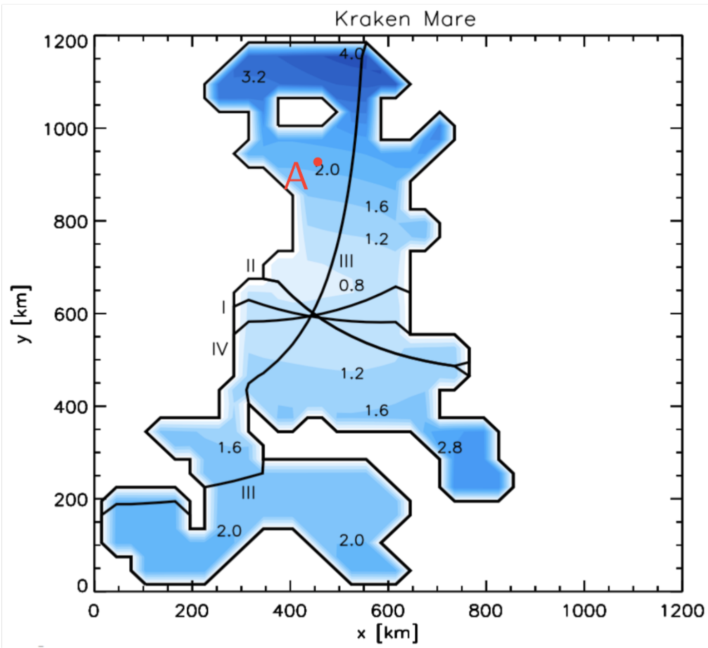
Solution: The solution is more easily seen through illustration:

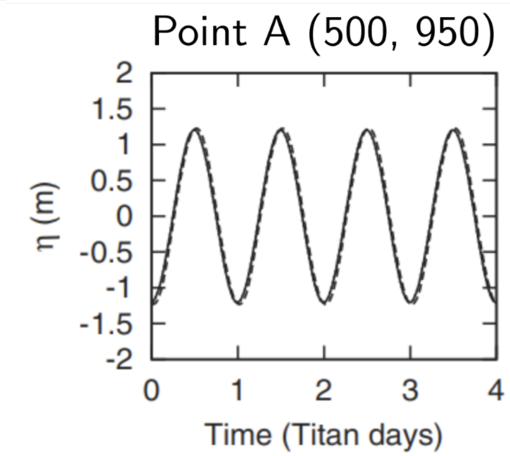


We can define the angle between the sub-focus point and the sub-Saturn point (thus the center of the tidal bulge) as θ. At apoapsis (point 1), $\theta = 0$ and Titan faces Saturn. Then, as Titan orbits, θ increases until it reaches its maximum at point 3. θ will then go back to 0 at periapsis and reach another maximum opposite to point 3 (not shown).

Thus, the period of the librational tide is half of the orbital period, or $\boxed{\text{8 days}}$. (7.5 days if (1) is incomplete)

Titan has several lakes composed of hydrocarbons like methane and ethane. The largest of these lakes, Kraken Mare, is located in the Northern Hemisphere and faces Saturn. Tokano (2010) performed a simulation of the tides generated in Kraken Mare, shown in the figure below. The bold lines are cotidal lines, while the shading shows the tidal range.





5. (2 points) In Titan and Earth days respectively, what is the time elapsed between two neighboring cotidal lines? (*Note:* cotidal lines are equally spaced in time)

Solution: We observe that the tides in Kraken Mare resemble amphidromic cells as seen on Earth. Cotidal lines (marking areas of the same tidal phase) rotate around the amphidromic point (though here, there is no true amphidromic point, because the tidal range is nonzero throughout the lake). There are eight cotidal lines which rotate with a period of 1 Titan day (see tide graph at A). Thus the time elapsed between two neighboring lines is 1/8 Titan days or 2 Earth days (15/8 Earth days if (1) is incomplete).

6. (1 point) The tide at Point A is:

   A. Semidiurnal

   B. Diurnal

   C. Mixed semidiurnal

   D. Mixed diurnal

Solution: The tidal period is a single Titan day.

7. (2 points) Titan is less massive than the Earth, meaning it has less gravitational pull. Despite this, Titan's surface atmospheric pressure is 50% higher than Earth. Briefly account for this observation.

Solution: Titan is much further from the Sun than Earth, meaning that its equilibrium temperature is much colder. Thus, gas particles in the atmosphere have less kinetic energy, preventing significant escape.

Section II: Problem 2

Question	1	2	3	4	5	6	Total
Points	2	2	3	3	3	2	15 (30%)

The early histories of Mars and Earth may have been quite similar (active hydrologic cycle, primitive atmosphere, etc.) before their paths diverged. One of the factors that contributed to this divergence is the disappearance of the Martian magnetic field.

1. (2 points) In addition to core convection, give another criterion necessary for a planetary body to maintain a magnetic field.

Solution: The fluid must be electrically conductive and the planetary body must be rotating. (Either is acceptable)

2. (2 points) Core convection may be maintained through thermally-driven instability or compositionally-driven instability. The latter may be dominant in small planetary bodies and is a result of the presence of lighter elements, namely sulfur, in the iron-dominated core. We will explore one hypothetical regime of compositionally driven convection.

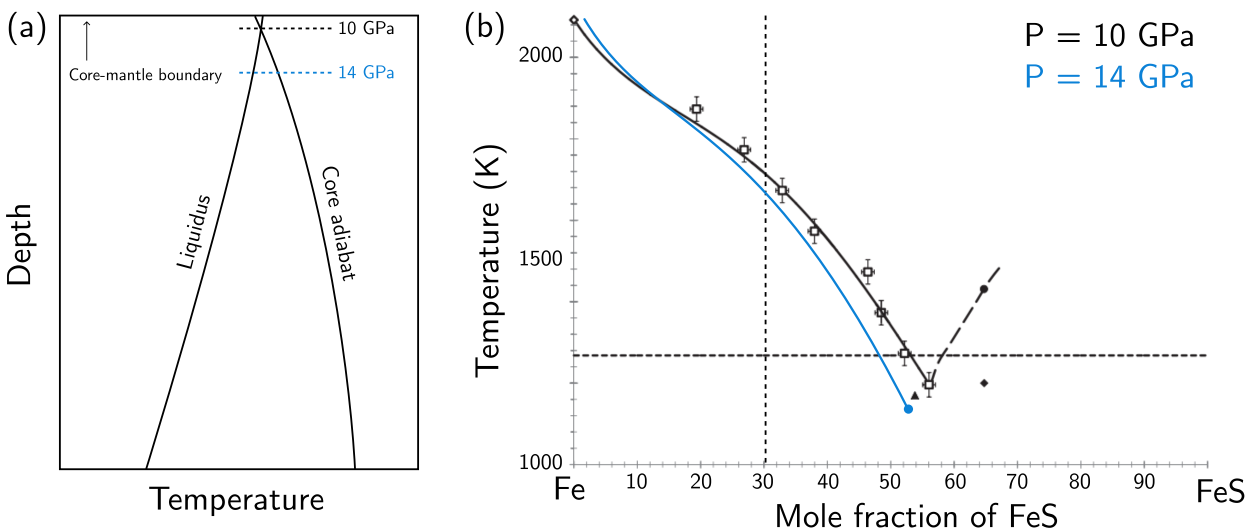


Figure 1: **(a)** a depth profile of an outer core. The core adiabat is the temperature of the outer core. Sulfur concentration is constant (30% FeS) with depth. **(b)** liquidus curves of the Fe-FeS system at two pressures.

Consider the outer core given by Figure 1(a). This will be the *initial* condition. If the core-mantle boundary (CMB), where the liquidus and the core adiabat intersect, is at a pressure of 10 GPa, which of the following is true?

I) At the constant sulfur concentration, the liquidus temperature decreases with increasing pressure.

II) Solid Fe will precipitate from the core-mantle boundary.

III) Solid FeS will precipitate from the core-mantle boundary.

A. II only

B. III only

C. II and III

D. I and II

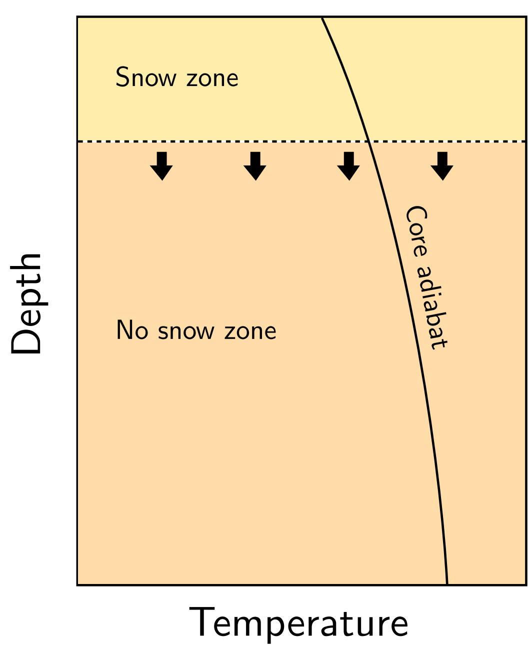
E. I and III

F. I, II, and III

Solution: From 1a, we see that the liquidus temperature decreases with increasing depth, and thus, pressure. From 1b, we see that the initial composition of the mantle is to the iron-rich side of the eutectic. Thus, iron will precipitate at the CMB.

3. (3 points) Let us now consider what will happen after this initial condition. Following the precipitation of solid, sulfur concentration will no longer be constant with depth. In terms of the phase change processes, describe how the sulfur concentration will vary with depth after the initial precipitation. (*Hint*: consider fractional crystallization)

Solution: Solid Fe will precipitate from the CMB, so the precipitation zone will be enriched in sulfur. Pure Fe is heavier than the iron-sulfur melt, so it will sink. Once it sinks to a depth where the core temperature is higher than the liquidus, it will melt once again and enrich the layer below with iron.



4. (3 points) Let's call the depths in which the solid is precipitating the "snow zone." The extent of the snow zone is ultimately controlled by the intersection of the adiabat with the liquidus. Considering your answer in 3, justify why the bottom limit of the snow zone grows deeper over time.

> **Solution:** Solid Fe enriches the liquid below the snow zone in iron once it remelts. This shifts the liquidus towards higher temperature (see 1b), until the liquidus reaches the core adiabat. Once this happens, iron precipitation at this new depth can occur, and the process repeats at lower depth. Hence, the snow zone grows over time. (Note that the liquidus does not exceed the core adiabat because iron precipitation effectively "buffers" the liquidus temperature)

5. (3 points) Where does compositionally driven instability trigger convection, the snow zone, no-snow zone, or both? Justify your answer.

> **Solution:** Compositionally-driven convection occurs in the no-snow zone. There, iron snowing from the snow zone remelts; iron is heavier than the iron-sulfur melt, so it sinks, generating convective motion. (In other words, iron is enriched from the top down). In the snow zone, there is a stable pressure gradient because sulfur is enriched from the top down (this does not generate instability).

6. (2 points) It has been hypothesized that Mars may have generated a magnetic field in its past through this mechanism. Considering your answer in 5, briefly explain why Mars no longer has an active magnetic field.

> **Solution:** The depth below the snow zone where convection can occur shrinks over time as the snow zone grows. Thus, the magnetic field is only active while there is a no-snow zone.

END OF EXAM