USESO 2022

Rocks & Minerals KEY

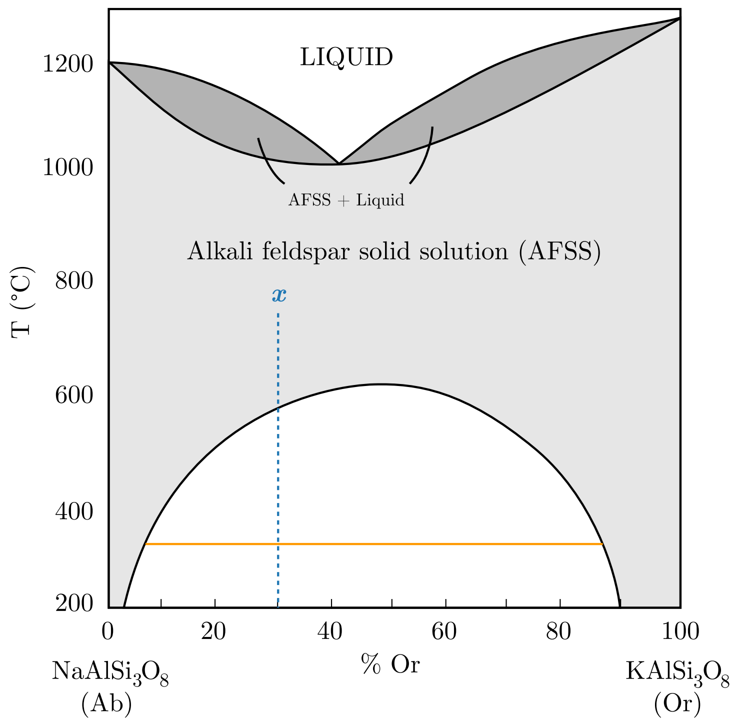
Instructions:

- There are three problems each worth 10 points; you have 30 minutes
- A calculator is allowed; show all work for calculations unless otherwise stated

Problem 1

Question	1	2	Total
Points	5	5	10 (33%)

The phase diagram below shows a large feldspar crystal of composition x as it cools along the dotted blue line. Ab and Or denote albite and orthoclase, respectively.



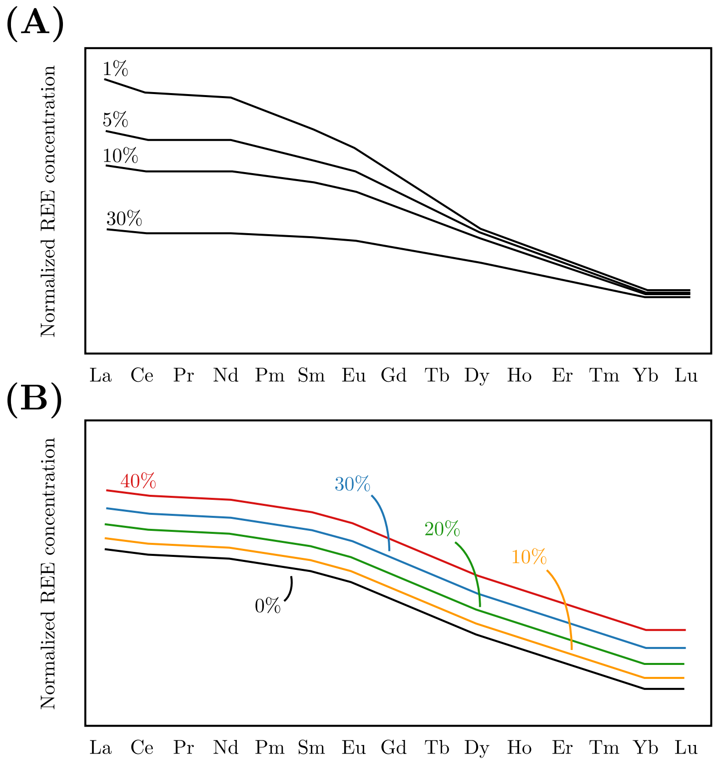
1. (a) (2 points) Explain what happens as the crystal cools below 580 °C. Briefly describe the texture that is formed by this process.

> **Solution:** As the crystal cools to below 580 °C, an albite-rich phase starts to exsolve (separate) from an orthoclase-rich phase. This forms a perthitic texture, where there are alternating bands of the whitish albite-phase and the pinkish orthoclase phase.

   (b) (3 points) Assuming that the crystal remains in chemical equilibrium, determine the compositions and relative proportion of all solid phases present at 330 °C (denoted as the orange line). Show your work and give your answer as percentages.

> **Solution:** At 300 °C, the compositions of the exsolved phases are 7% Ab and 87% Ab. We can then use a weighted average (the so-called lever rule) to calculate the relative proportion of each phase.
>
> $$(0.07)(1 - x) + (0.87)x = 0.30$$
>
> Solving for x, we find that the Or-dominant phase makes up 29% and the Ab-dominant phase makes up 71%.

The two diagrams below show the relative content of each rare earth element (REE) in two magmas undergoing differentiation. REEs are relatively incompatible, meaning that they do not fit well into the crystal structure of most minerals.



2. (a) (2 points) Differentiation shown in A and B have been caused either by fractional crystallization or partial melting. The percentages above each line show the degree to which either of these processes have taken place. Match A and B to either fractional crystallization or partial melting and briefly explain your choice.

Solution: A - partial melting, B - fractional crystallization. In diagram A, progression of the differentation leads to *decreasing* amounts of incompatible REEs in the magma. The initial magma is enriched in REEs, but REE concentration decreases as more melt forms; this occurs during partial melting. In diagram B, progression of the differentation leads to *increasing* amounts of incompatible REEs in the magma. This is indicative of fractional crystallization because incompatible elements tend to stay in magma during crystallization.

(b) (3 points) As the elements trend from Lanthanum (La) to Lutetium (Lu), do they become more or less incompatible? Justify your answer using the diagram(s). Then, use geochemical concepts to hypothesize why this trend exists.

Solution: They become less incompatible. In all of the magmas shown in the diagrams above, their normalized concentration decreases from left to right, indicating that lanthanides closer to Lu tend to remain in crystallized phases during magma differentiation. Compatibility is typically determined by valency and ionic radius. The trend that we see here is caused by the decreasing trend of ionic radius across the lanthanides. (Further, it is unrealistic to see a continuous trend in compatibility caused solely by differences in valency. In fact, all lanthanides in geologic settings are 3+ with a couple exceptions.)

Problem 2

Question	1	2	3	4	Total
Points	3	4	2	1	10 (33%)

The age of the Earth was constrained in the 1950s using a technique called lead-lead dating on meteorite samples. This method relies on measuring the ratio of radiogenic lead isotopes ^{206}Pb and ^{207}Pb to a non-radiogenic reference isotope ^{204}Pb. The two relevant decay pathways are:

$$^{238}\text{U} \longrightarrow {}^{206}\text{Pb}$$
$$^{235}\text{U} \longrightarrow {}^{207}\text{Pb}$$

1. (3 points) Write an equation relating a sample's current ratio of $^{206}\text{Pb}/^{204}\text{Pb}$ to its age T, the current ratio of $^{238}\text{U}/^{204}\text{Pb}$, and the half-life of ^{238}U, denoted as $t_{1/2}$. You may assume that the sample is isolated, but the original ^{206}Pb concentration is **not** zero. (*Hint: the concentration of ^{204}Pb remains constant with time.*)

Solution: Begin with accounting for the original and radiogenic ^{206}Pb:

$$^{206}\text{Pb} = {}^{206}\text{Pb}_0 + \left({}^{238}\text{U}_0 - {}^{238}\text{U}\right)$$

Since $^{238}\text{U} = {}^{238}\text{U}_0 \left(2^{-T/t_{1/2}}\right)$, then we can rearrange the first equation to

$$^{206}\text{Pb} = {}^{206}\text{Pb}_0 + {}^{238}\text{U}\left(2^{T/t_{1/2}} - 1\right)$$

We then divide both sides by ^{204}Pb to obtain:

$$\frac{^{206}\text{Pb}}{^{204}\text{Pb}} = \frac{^{206}\text{Pb}_0}{^{204}\text{Pb}} + \frac{^{238}\text{U}}{^{204}\text{Pb}}\left(2^{T/t_{1/2}} - 1\right)$$

This dating method also assumes that the ratio of $^{238}\text{U}/^{235}\text{U}$, denoted r, is the same as that of Earth ($r = 137.8$). For abbreviation, from here we will denote $^{206}\text{Pb}/^{204}\text{Pb}$ as R_{206} and $^{207}\text{Pb}/^{204}\text{Pb}$ as R_{207}.

2. (4 points) Show that for an isolated sample, the two lead isotope ratios can be related by:

$$R_{207} = R_{206}\left[\frac{2^{T/t_{235}} - 1}{r\left(2^{T/t_{238}} - 1\right)}\right] + k$$

Where t_{235} and t_{238} are the half-lives of ^{235}U and ^{238}U, respectively, and k is a constant.

Solution: Begin by writing the two decay equations for R_{207} and R_{206}:

$$R_{207} = \frac{^{207}\text{Pb}_0}{^{204}\text{Pb}} + \frac{^{235}\text{U}}{^{204}\text{Pb}}\left(2^{T/t_{235}} - 1\right)$$

$$R_{206} = \frac{^{206}\text{Pb}_0}{^{204}\text{Pb}} + \frac{^{238}\text{U}}{^{204}\text{Pb}}\left(2^{T/t_{238}} - 1\right)$$

We can rewrite the second term in the equation for R_{207} in terms of the second term in the equation for R_{206}:

$$R_{207} = \frac{^{207}\text{Pb}_0}{^{204}\text{Pb}} + \left(R_{206} - \frac{^{206}\text{Pb}_0}{^{204}\text{Pb}}\right)\left[\frac{2^{T/t_{235}} - 1}{r\left(2^{T/t_{238}} - 1\right)}\right]$$

Distributing the factor,

$$R_{207} = \frac{^{207}\text{Pb}_0}{^{204}\text{Pb}} + R_{206}\left[\frac{2^{T/t_{235}} - 1}{r\left(2^{T/t_{238}} - 1\right)}\right] - \frac{^{206}\text{Pb}_0}{^{204}\text{Pb}}\left[\frac{2^{T/t_{235}} - 1}{r\left(2^{T/t_{238}} - 1\right)}\right]$$

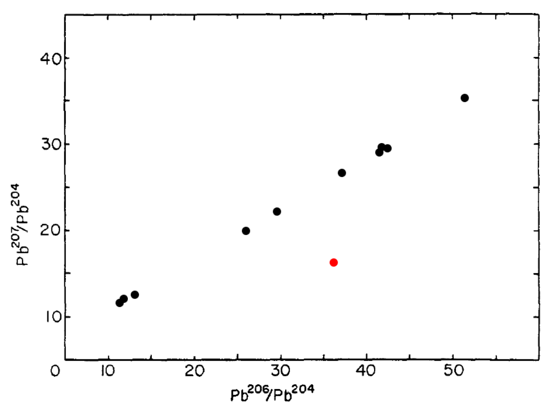
Notice that the first and last terms can be grouped into k, which is a constant that depends on the age of the sample and initial (unknown) concentrations of ^{207}Pb and ^{206}Pb:

$$R_{207} = R_{206}\left[\frac{2^{T/t_{235}} - 1}{r\left(2^{T/t_{238}} - 1\right)}\right] + k$$

$$k = \frac{^{207}\text{Pb}_0}{^{204}\text{Pb}} - \frac{^{206}\text{Pb}_0}{^{204}\text{Pb}}\left[\frac{2^{T/t_{235}} - 1}{r\left(2^{T/t_{238}} - 1\right)}\right]$$

We have shown that when the *current* ratios R_{207} and R_{206} for samples of the same age T are plotted, the graph will be linear with a slope that depends only on the age of the samples. This is the so-called isochron dating method, which is useful because it does not require knowledge of any initial concentrations.

3. (2 points) 10 meteorite samples from various localities are analyzed and their lead compositions are plotted below **in black**. Assuming these meteorites formed concurrently with Earth, **explain** how the data from this plot can be used to estimate the age of the Earth. You **do not** need to perform any calculations or algebra.



Solution: As seen in the equation in question 2, the linear relationship between R_{207} and R_{206} for samples of the same age means that the slope m is equal to:

$$m = \left[\frac{2^{T/t_{235}} - 1}{r\left(2^{T/t_{238}} - 1\right)}\right]$$

Since the slope depends only on the age of the samples and other known constants, the age of the Earth can be estimated by determining the slope of the isochron plot above and solving for T. (If we were to do the calculation, we would obtain a value of $T \approx 4.5 \times 10^9$ yr.)

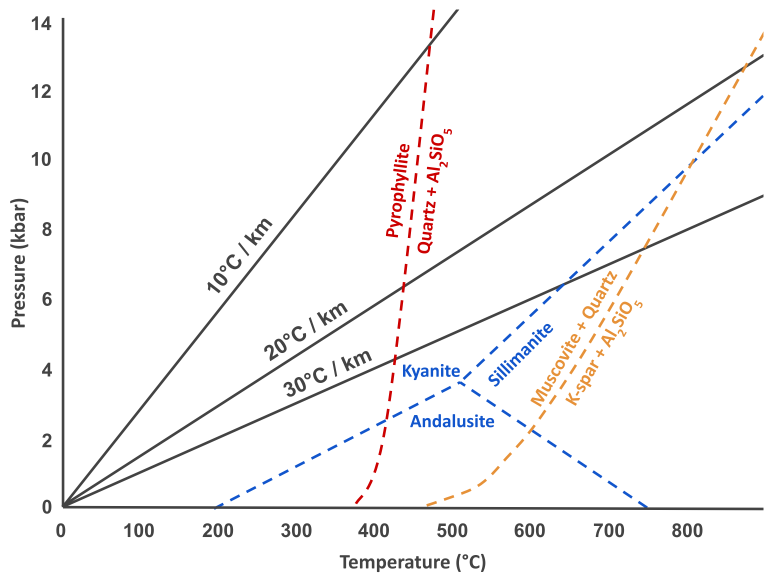
4. (1 point) The graph of $^{207}\text{Pb}/^{204}\text{Pb}$ and $^{206}\text{Pb}/^{204}\text{Pb}$ for the 10 meteorite samples is linear. However, when a sample from a Martian meteorite is analyzed and plotted in red, it falls off of the line. Propose one reason for this.

Solution: It is a different age, since Martian meteorites can be ejected from Mars after the formation of the solar system.

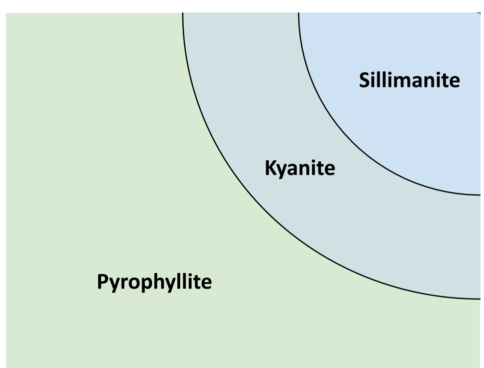
Problem 3

Question	1	2	3	4	Total
Points	1	3	2	4	10 (33.3%)

The diagram below shows stability ranges for minerals at various metamorphic grades. Andalusite, kyanite, and sillimanite are polymorphs with the same chemical formula, Al_2SiO_5.



A geologist has identified the presence of pyrophyllite, kyanite, and sillimanite at various points in the metamorphic complex shown below. While these are not the only minerals present, they offer useful information about the complex and its formation.



1. (1 point) Which of the following most likely describes the sequence of metamorphic facies seen in this complex?
   A. Blueschist to amphibolite
   B. Prehnite-pumpellyite to eclogite
   C. Greenschist to granulite
   D. Zeolite to hornfels

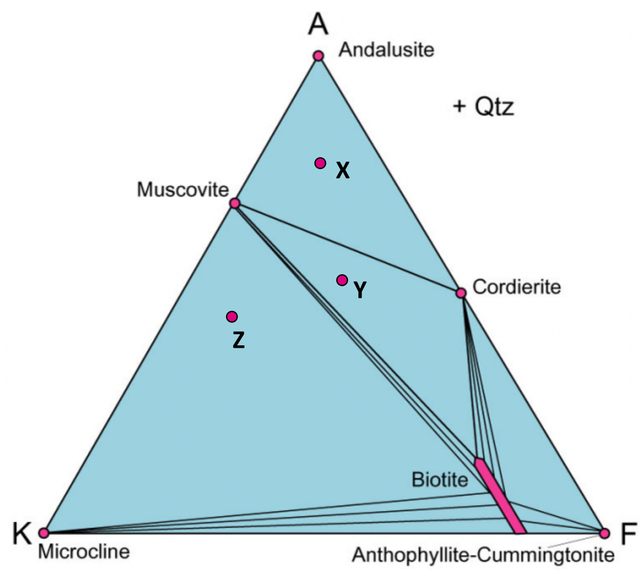
2. (a) (2 points) Explain the significance of the production of H_2O in the environment where this reaction occurs.

Solution: When dehydration reactions occur during metamorphism, they produce water and other volatiles which are forced out of the rock by the high pressure. The addition of volatiles lowers the mantle solidus, allowing limited partial melting in the region where these reactions occur.

(b) (1 point) Minerals like kyanite and sillimanite are not stable at surface conditions, but remain in the complex anyway. What conditions would be required for them to convert back to a more stable form?

Solution: Because these reactions involve the release of water, their reverse reactions require the addition of water. This typically occurs with an influx of hydrothermal fluids.

The diagram below is a triangular AKF diagram showing various common metamorphic minerals. The components A, K and F are defined as follows:



A = $[Al_2O_3 + Fe_2O_3]$ - $[Na_2O + K_2O + CaO]$
K = $[K_2O]$
F = $[FeO + MgO + MnO]$

3. (2 points) Where would pyrophyllite, $Al_2Si_4O_{10}(OH)_2$, be located on this diagram?

   A. At point A

   B. At point K

   C. At point F

   D. Between points A and K

   E. Between points K and F

   F. Between points A and F

Solution: We can rewrite the formula of pyrophyllite in terms of "constituent oxides" as $(Al_2O_3)(SiO_2)_4(H_2O)$. Pyrophyllite has an Al_2O_3 component but no K, Fe, Mg, or Mn oxides, so it has no K or F component.

4. (a) (2 points) Which of the points labeled on the diagram would most likely produce the composition seen in the complex?

A. X

B. Y

C. Z

Solution: The complex has 3 minerals of high aluminum content, so we can reason that the closest point to A, X is most likely the answer. Y and Z wouldn't form pyrophyllite at lower temperatures and pressures, and would instead form muscovite, so the answer could not be Y or Z.

(b) (2 points) Using the stability chart, explain how the complex would differ if its parent rock had a composition defined by either of the other two points.

Solution: Points Y and Z both have compositions that include muscovite but not pyrophyllite. Because muscovite is stable at a higher grade than pyrophyllite, Al_2SiO_5 will form later in metamorphism at the orange boundary labeled on the chart. In fact, because this occurs so late, the complex will have little to no kyanite; Al_2SiO_5 will not form until sillimanite is the stable phase.