



USESO 2026

Training Camp Exam

Section II

KEY

Instructions:

- Section II consists of 5 multipart problems that further assess geoscience knowledge in the form of free-response and multiple choice questions.
- A calculator is allowed. Show all work for calculations.
- Any space on the page may be used for scratch paper, but only work on your Answer Sheet will be graded.
- Print your **USESO Student ID** on every page of the Answer Sheet.

Problem 1

Question	1	2	3	4	Total
Points	10	1	4	5	20 (20%)

This problem examines various aspects of Earth's interior.

- The depth of magma formation can be inferred from the signatures of rare earth elements (REEs) in aluminous minerals.

- (1 point) The table below shows the properties of three selected aluminous minerals at depth in the mantle. Order the three minerals by increasing depth at which they would typically be found. **Briefly justify** your ordering.

Mineral	Chemical Formula	Substitution Sites	Density (g/cm ³)	P-wave Velocity (km/s)
Spinel	(Mg, Fe)(Al, Cr) ₂ O ₄	(Al ³⁺ , Cr ³⁺)	3.6	9
Garnet	(Mg, Fe, Ca) ₃ Al ₂ Si ₃ O ₁₂	(Mg ²⁺ , Fe ²⁺ , Ca ²⁺)	4.0	10
Plagioclase	CaAl ₂ Si ₂ O ₈	Ca ²⁺	2.7	7

Solution: Plagioclase, spinel, garnet. This follows from increased density trends (greater depth phases are typically compressed into more tightly packed crystals). It also follows from P-wave velocity (greater depth phases are typically more rigid).

- The tendency of each REE to be incorporated into a given mineral, known as compatibility, can be expressed by its mineral-specific partition coefficient P , shown in Figure 1.

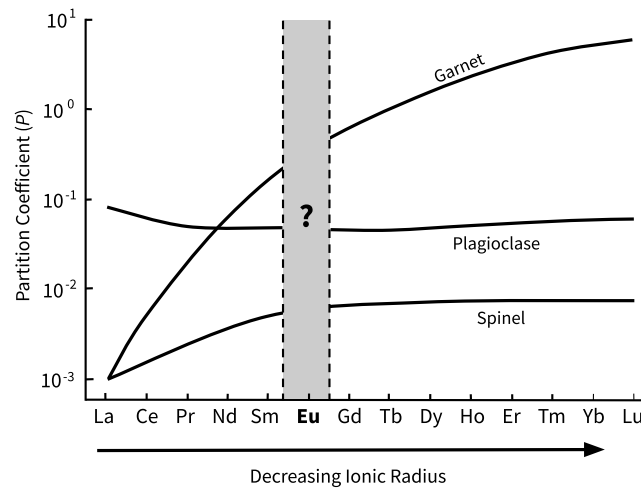


Figure 1: Mineral-specific partition coefficients (P) across the lanthanides for three selected aluminous minerals. Displayed coefficients per element are an average over all oxidation states. The partition coefficients for Europium are not shown.

P is used mathematically in the following equation:

$$\frac{C_L}{C_0} = \frac{1}{P(1 - F) + F}$$

where C_L is the concentration of the element in the melt, C_0 is the concentration of the element in the melt's source rock, and F is the melt fraction.

- i. (2 points) Does low or high P indicate that an element is highly incompatible? **Briefly explain.** *Hint: Use the provided equation in your reasoning.*

Solution: Low P indicates high incompatibility. In the case of very slight melting, F is very small. If an element is highly incompatible, we expect a high concentration in the melt with little left behind in the solid. This means the values on both sides of the equation to be large. For a small F , the right side equation is approximately $\frac{1}{P}$. For this to be large, P is small.

- ii. (2 points) Using your answers to 1(a) and 1(b)i, **explain** whether elements with smaller ionic radii are relatively enriched or depleted in partial melts derived from a deep source.

Solution: From 1(a), we know that garnet is likely present in the source rock of deep melts. From Figure 1, we see that garnet has a much higher partition coefficient for smaller incompatible elements than spinel or plagioclase. Thus we would expect the garnet to “hold on” to these elements during melt formation, causing them to be depleted in the resulting partial melts.

- (c) While most REEs are commonly found only in their 3+ oxidation states, europium is unique because it can often exist in both 2+ and 3+ oxidation states, with the 2+ ion having a significantly larger ionic radius than the 3+ ion. This unique property gives rise to an anomalously high partition coefficient for europium compared to other REEs in certain minerals, known as the “europium anomaly.”

- i. (3 points) For which of the graphed minerals would you expect there to be a europium anomaly? For each mineral, **explain** why or why not.

Solution: For a europium anomaly to occur, the 2+ state of europium must be compatible to cause a general increase in europium compatibility. Spinel does not have a 2+ site to accept Eu^{2+} , so we do not expect there to be an anomaly. Garnet and plagioclase both have 2+ sites, but as seen in Figure 1, ions with particularly large radii have very low partition coefficients in garnet. Since Eu^{2+} has an abnormally large radius, it would be incompatible in garnet. Only plagioclase would display an anomaly.

- ii. (2 points) **Briefly describe** the formation of the lunar highlands and maria. Would you expect the lunar maria to be enriched, depleted, or comparable in europium to the bulk composition of Earth upon formation? **Justify** your response.

Solution: The lunar highlands formed when semi-solid plagioclase rose up to the surface. Afterwards, the maria formed from melts deep in the Moon’s interior. Since the lunar highlands brought europium-rich plagioclase up to the surface, we expect that the reservoir left behind would be relatively depleted in europium. As maria are then made from this reservoir, we expect them to be depleted.

2. (1 point) Scientists recently analyzed diopside grains from Hawaiian basalts and found that europium was more concentrated at the edges of the grains compared to their centers. This anomaly is unusual due to the diopside’s relatively low partition coefficient for europium. To understand this observation, they considered the diffusion coefficient (D) for various elements in diopside, defined as the rate at which elements diffuse from regions of higher to lower concentration.

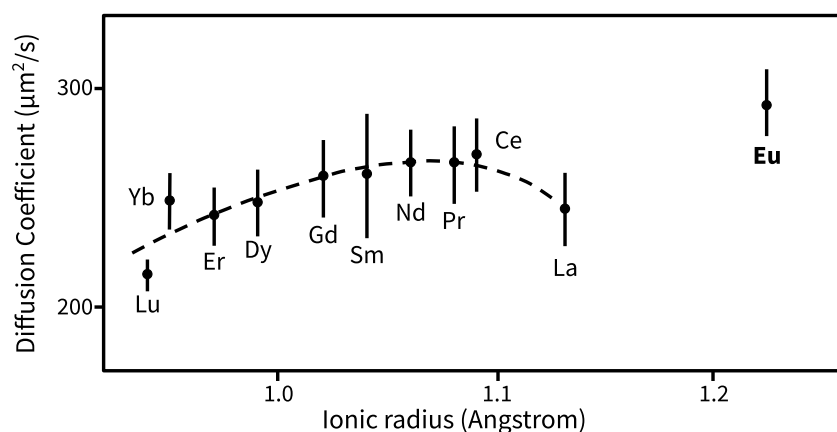
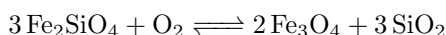


Figure 2: Diffusion coefficients for selected rare earth elements in diopside plotted against ionic radius.

Based on Figure 2, **propose** a likely explanation for the scientists' observation.

Solution: Because europium is not favorable within diopside's crystal structure, we expect that it stays relatively concentrated in the melt. However, due to its high diffusion coefficient compared to other REEs, it should be able to diffuse into the crystal structure faster than other elements. This causes a europium anomaly since europium becomes more concentrated compared to other REEs. Due to its outside-in diffusive nature, the concentration at the edges also makes sense.

3. The Fayalite-Magnetite-Quartz (FMQ) buffer is a standard reference for the redox conditions in igneous rocks. In this buffer, fayalite (Fe-rich olivine) reacts with oxygen to form magnetite and quartz, as shown below.



One way to quantify the state of this buffer is ΔFMQ , where a represents the effective concentration of a given mineral within the solid as in the equation below.

$$\Delta\text{FMQ} = \log_{10} \left(\frac{a_{\text{Mt}}^2 \cdot a_{\text{Qz}}^3}{a_{\text{Fa}}^3} \right)$$

- (a) (2 points) Does ΔFMQ of the oceanic crust generally increase or decrease with distance from the ridge axis? **Explain.**

Solution: Increase. The older oceanic crust far from the ridge has had more time to interact with the relatively oxygen-rich seawater. This oxidizes the constituent rocks, pushing the buffer to the right, increasing ΔFMQ .

- (b) (2 points) Using your answer to 3(a) and the information provided in 1(c), would subduction zone plagioclase show a stronger or weaker europium anomaly compared to normal mantle plagioclase? **Justify** your answer.

Solution: Due to their distance from mid-ocean ridges, subduction zones are relatively oxidized. This causes europium ions to be more often found in the 3+ oxidation state than the 2+ state. This dampens the europium anomaly since it is caused by the presence of Eu^{2+} ions.

4. The primary method of deformation within the Earth varies significantly with depth.

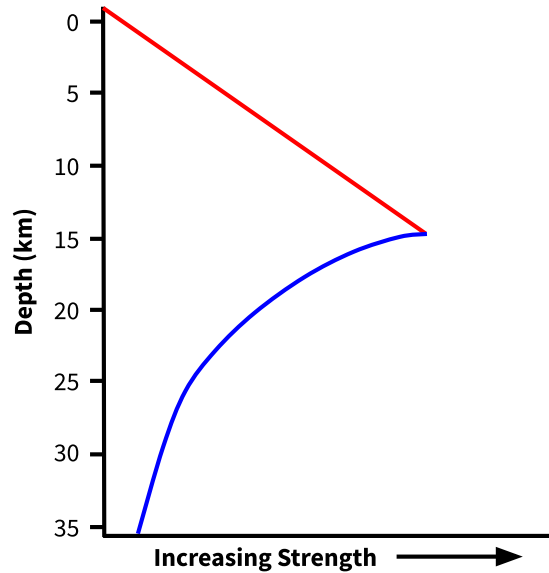


Figure 3: A piecewise graph of rock maximum tolerable stress before deformation as a function of depth, colloquially known as the “Christmas tree” diagram.

- (a) (1 point) **Identify** the primary environmental factor that drives the transition between the red and blue curves.

Solution: Temperature. At higher temperatures, the primary mode of deformation transitions from brittle to ductile deformation.

- (b) (1 point) **Explain** why the red curve displays increasing strength with increasing depth.

Solution: Pressure increases with depth. This squeezes rock together, increasing the frictional force that prevents failure modes like slippage or fracture.

- (c) (1 point) Based on Figure 3, at what depth do the strongest earthquakes occur? Round your answer to the nearest five kilometers and **briefly explain**.

Solution: 15 km. It is where the rock is at its strongest and can build up energy in the form of stress before it is suddenly released.

- (d) (2 points) Eclogitization has been cited as a mechanism for explaining extremely deep earthquakes, which are otherwise considered unlikely in an olivine-dominated mantle. Suppose that eclogite and an all-olivine rock have the same ductile deformation curve in Figure 3. Would eclogite strengthen with increasing depth more rapidly or more slowly than the all-olivine rock at shallow depths? **Explain**.

Solution: The deepest earthquakes occur at the deepest section of the brittle deformation curve (since they don't occur under ductile deformation conditions). Thus the two curves must intersect deeper. The red curve must be steeper in order to intersect deeper. This corresponds with eclogite strengthening more slowly than the all-olivine rock.

Problem 2

Question	1	2	3	4	5	Total
Points	1	4	2	7	6	20 (20%)

This problem explores several aspects of circulation over the Sahara.

- (1 point) The Saharan Air Layer (SAL) is a dry and dust-laden layer of the troposphere that forms over the Saharan Desert. One characteristic of the SAL is the presence of Saharan dust. What would be the net effect of these aerosols on the temperature at and below the SAL, respectively?

- A. Increase, increase
- B. Increase, decrease**
- C. Decrease, increase
- D. Decrease, decrease

Solution: Aerosols that sit in the atmosphere increase absorbance of light. This heats the air at the level of the aerosols and blocks energy from reaching the air below. Thus the answer is B.

- The African Easterly Jet (AEJ) is a fast-flowing, easterly wind current in the lower troposphere that has a similar mechanism to the Northern Hemisphere's polar jet stream. The AEJ blows the SAL westward off the continent and over the Atlantic.

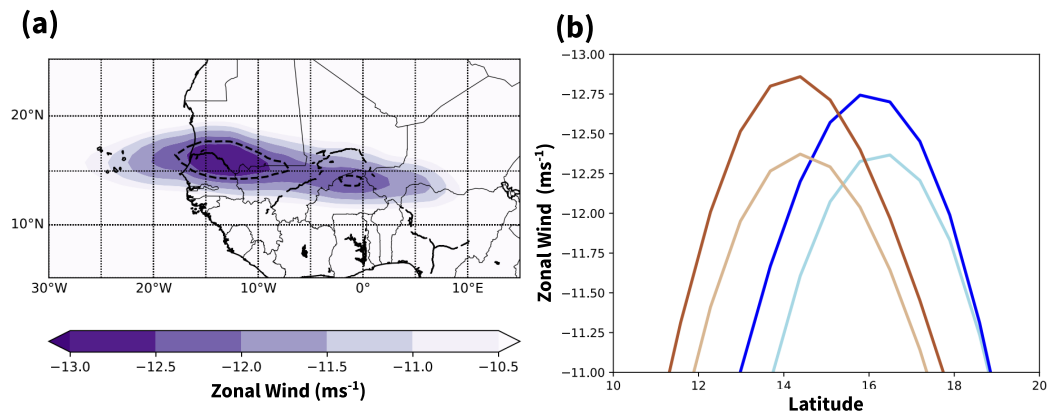


Figure 1: (a) July–September time-averaged zonal (i.e., east-west) wind composite at the altitude of the AEJ core in a particular year. (b) July–September time-averaged zonal wind profiles at the height of the AEJ for four years.

- (1 point) **Briefly explain** why the AEJ blows from east to west.

Solution: Warmer air over the Sahara and cool air from tropical rain causes a north to south pressure gradient aloft. The Coriolis force turns this clockwise, causing it to be an easterly wind.

- (b) (1 point) Figure 1(b) contains four curves corresponding to four “types” of years: wetter & less dusty (WLD), wetter & more dusty (WMD), drier & less dusty (DLD), and drier & more dusty (DMD). A relatively dustier year occurs due to higher SAL dust content, and a relatively wetter year is characterized by a broader latitudinal extent of tropical African precipitation. **Identify** which curve corresponds to each type.

Solution: Blue: wetter and more dusty; Cyan: wetter and less dusty; Dark brown: dryer and more dusty; Light brown: dryer and less dusty.

- (c) (2 points) **Explain** your reasoning for part (b).

Solution: A broader latitudinal extent of tropical precipitation moves the low aloft pressure further north. This would push the corresponding jet further north. The blue and cyan curves correspond to wetter years. Increased dust strengthens absorbance in the north → increased warm anomaly → increased temperature gradient → increased jet strength. The blue and dark brown curves correspond to dustier years.

3. (2 points) Researchers have found that a large amount of the dust within the SAL was entrained by unique behavior associated with Saharan thunderstorms. Due to high temperatures and low humidity, many Saharan thunderstorms produce virga, rain that evaporates before reaching the ground. **Explain** how virga can create strong, dust-entraining surface winds.

Solution: As rain falls and evaporates, it absorbs a large amount of heat as latent heat. This significantly cools the surrounding air, creating an anomalously cold air parcel. This parcel rapidly sinks to the ground, then spreads out into strong, dust-entraining surface winds.

4. In West Africa, the monsoon season begins in April as intense rainfall hugs the Guinean coast (Figure 2). This coastal precipitation abruptly shuts off toward the end of June, when the peak precipitation moves inland toward the Sahel (near 10°N) in a discontinuous shift known as the monsoon jump.

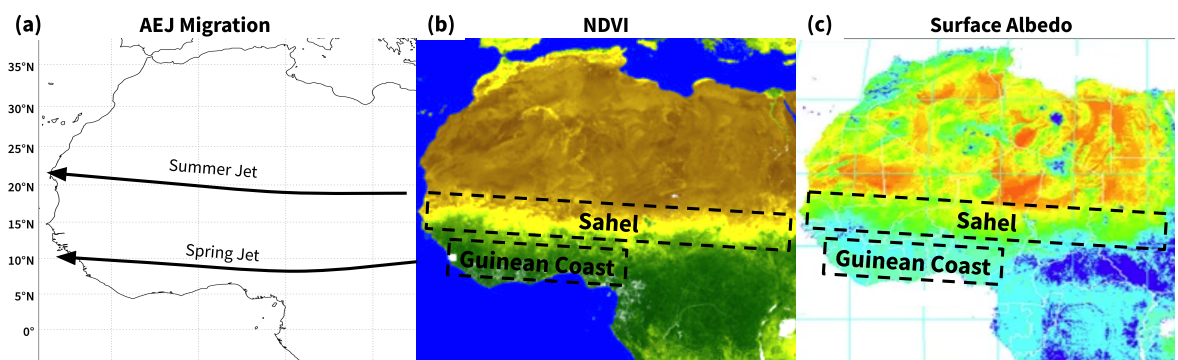


Figure 2: Three maps of Western Africa with indicated relevant regions. (a) Mapping of seasonal migration of the AEJ. (b) Normalized Difference Vegetation Index (NDVI) values where green indicates high levels of vegetation. (c) Surface albedo where red areas indicate higher albedo values.

- (a) (2 points) Weeks before the monsoon jump, a shallow circulation cell exists over the Sahel but does not create much precipitation. Using the figures 2(b) and (c), **explain** the cause of this circulation **and** why it remains confined near the surface.

Solution: The Sahel is a medium albedo region, which allows it to absorb more energy than the desert to the north. This absorption provides the energy to drive a circulation cell. The low NDVI shown in (b) shows a lack of vegetation and thus moisture in the region. Circulation is deepened by latent heat release during condensation, driving further lifting. Since the Sahel doesn't have much moisture this process is missing and circulation remains shallow.

- (b) (1 point) The monsoon jump is driven by inertial instability, a condition that occurs when $f - \frac{\Delta u}{\Delta y} < 0$, where $\frac{\Delta u}{\Delta y}$ is the northward gradient of westerly wind speed and f is a positive proxy for the strength of the Coriolis force. Using this information, **explain** why it is relatively easy for inertial instability to form along the Guinean coast.

Solution: f is very small along the Guinean coast due to its location (equatorial regions experience weak Coriolis force). This means that $\frac{\Delta u}{\Delta y}$ does not need to be very large to satisfy the inequality.

- (c) (1.5 points) Using Figure 2, **explain** why Guinean coastal air is inertially stable in the spring.

Solution: Because the AEJ flows easterly, it has a negative gradient in its westerly wind speed as you move north toward its strongest point. In the spring, this creates a negative value for $\frac{\Delta u}{\Delta y}$ along the Guinean coast. Since f is positive, the expression on the left is always positive and inertial stability occurs.

- (d) Separate from the climatic variation described in 2(a), the position of the AEJ also varies seasonally and migrates northward during summer.

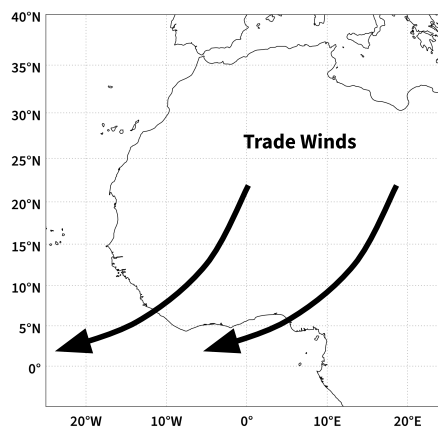


Figure 3: An idealized map of trade wind behavior over western Africa.

- i. (1.5 points) Using the wind pattern depicted in Figure 3, **explain** how this seasonal migration leads to a transition into inertial instability for air over the Guinean coast.

Solution: When the jet migrates north, the negative $\frac{\Delta u}{\Delta y}$ is no longer in place. This allows small positive perturbations in $\frac{\Delta u}{\Delta y}$ to trigger inertially unstable conditions.

- ii. (1 point) Inertial instability in summer leads to surface air parcels rapidly moving away from the region of instability. **Explain** how this results in the abrupt shutoff of precipitation along the Guinean coast associated with the monsoon jump.

Solution: Divergence at the surface causes the sinking of air, inhibiting precipitation in the area.

5. In the remainder of this problem, you will derive an expression for the speed c of an internal gravity wave at the boundary between the SAL and the underlying air by analogy to shallow surface gravity waves at the interface of Earth's atmosphere and oceans. We will assume the wave behaves as a shallow water waves with no variation along the z -direction (Figure 4).

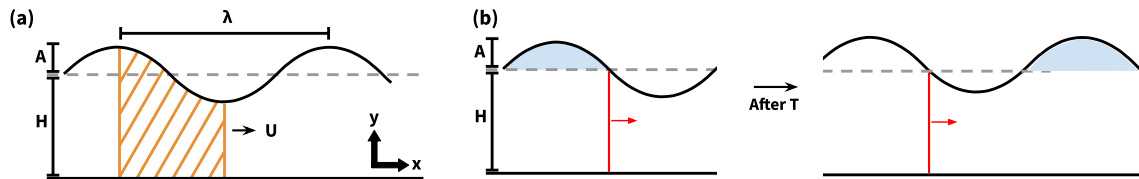


Figure 4: (a) Surface gravity wave at ocean-atmosphere interface with height H , amplitude A , wavelength λ , and velocity U labeled. Wave speed c is defined as λ/T . Assume water has density ρ and air has no density. The orange shading represents a specific water parcel to which we will apply momentum balance. (b) Evolution of the surface gravity wave after period T .

- (a) (1 point) We can derive an expression for c via mass balance. Referring to Figure 4(b), **fill in the following blanks** with the appropriate variables:

By comparing the wave shape before and after time T , the mass loss through the red profile due to the movement of the shaded portion of the wave over timescale T is proportional to $A \cdot \lambda \cdot \rho$.

We can also estimate this mass loss in a different way: the mass transported out of the left portion of the wave over time T is proportional to $T \cdot H \cdot \rho \cdot$ _____.

By equating these two expressions, we derive $c \propto H \cdot U /$ _____.

Solution: U , A . Mass transported over time is given by cross sectional area $\cdot$ velocity $\cdot$ time $\cdot$ density. Everything is represented already except velocity U . Equating $T \cdot H \cdot \rho \cdot U = A \cdot \lambda \cdot \rho$ and solving for $\lambda/T = c$ gives $H \cdot U/A$.

- (b) We can derive a second expression for c by applying horizontal momentum balance. Assume that the water velocity is independent of depth and that the pressure field is determined solely by the weight of the overlying water.

- i. (1 point) **Identify** which side (left or right) of the orange parcel in Figure 4(a) experiences a greater average pressure. Hence, **identify** the direction of the net pressure force on the parcel. **Briefly justify** your answer.

Solution: Left. There is a higher column of water to cause pressure further down in the water column. This higher pressure acts in all directions so it will push the parcel right.

- ii. (1 point) Referring to Figure 4(a), **fill in the following blanks** with the appropriate variables:

Let Δp denote the pressure difference between the left and right sides of the orange parcel. The net horizontal force acting on the parcel is proportional to $\Delta p \cdot H$. Furthermore, Δp is proportional to $\rho \cdot g \cdot$ _____, where g is the gravitational acceleration.

We can obtain a separate expression for the net horizontal force acting on the orange parcel using Newton's second law. Assuming the characteristic acceleration of the parcel is proportional to U/T and mass is proportional to $H \cdot \lambda \cdot \rho$, the net horizontal force can be expressed as $H \cdot \lambda \cdot \rho \cdot U/T$.

By equating these two expressions, we derive $c \propto g \cdot A /$ _____.

Solution: A, U . Differences in water height cause differences in water pressure at all heights along the sides of the parcel. The magnitude of this difference depends on the difference in water depth at different horizontal levels along the parcel. This difference in depth is given by A . Setting $\rho \cdot g \cdot A \cdot H = H \cdot \lambda \cdot \rho \cdot U/T$, we again solve for $\lambda/T = c$ and get $c \propto g \cdot A/U$.

- (c) (2 points) By equating the expressions for c derived in parts 5(a) and 5(b)(ii), **derive** a proportionality relationship for c in terms of a single variable and any necessary physical constants. **Show your work.**

Solution: $U \propto \frac{Ac}{H}$ and $U \propto \frac{gA}{c}$. Setting these equal and solving for c , we get $c \propto \sqrt{gH}$

- (d) (1 point) In the case of an internal gravity wave between two arbitrary air masses, we substitute g' for g in all of the above derivations. Would you expect g' to be smaller or larger than g ? **Briefly explain.**

Solution: Smaller. g' represents the effective force of gravity caused by density differences. When these are smaller (like in an atmospheric setting), g' will be smaller.

Problem 3

Question	1	2	3	4	5	Total
Points	3	2	7	4	4	20 (20%)

This problem considers how various processes affect circulation in the Atlantic and Pacific Oceans.

1. The positive phase of the North Atlantic Oscillation (NAO) is associated with an increased pressure difference between the Azores High and Icelandic Low, shown in Figure 1.

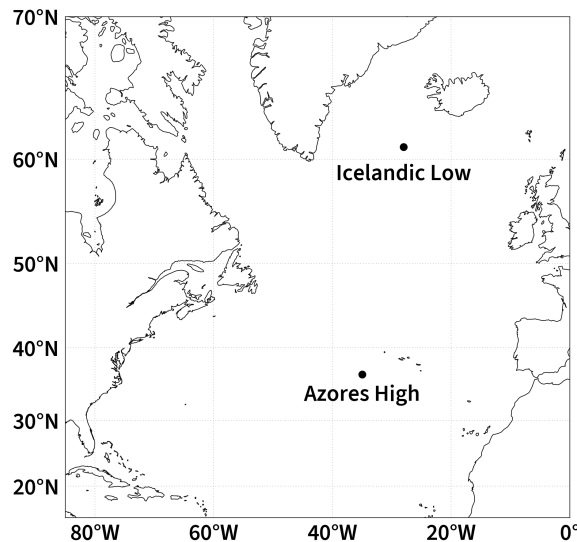


Figure 1: Locations of the Azores High and Icelandic Low.

- (a) (2 points) **Describe** the wind patterns expected during positive NAO compared to normal conditions. How does this affect the heat budget of North Atlantic water currents?

Solution: The NAO strengthens surface westerly winds, which leads to increased evaporative cooling and decreased heat carried by North Atlantic currents.

- (b) (1 point) The Atlantic Meridional Overturning Circulation, or AMOC, refers to the northern branch of Atlantic thermohaline circulation. Given your answer to part (a), how would a positive NAO affect the strength of AMOC? **Briefly explain.**

Solution: Since positive NAO cools the North Atlantic, it increases surface water density, creating more favorable conditions for the sinking of water. This strengthens the AMOC.

2. The Stommel box model, shown in Figure 2, is a simplified model of AMOC in the Northern Hemisphere.

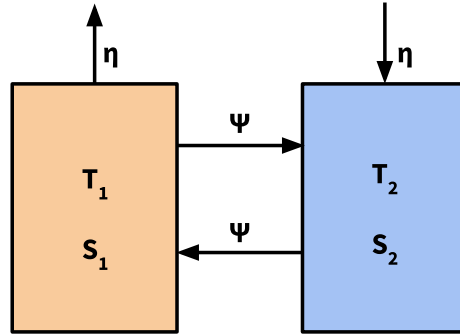


Figure 2: Simplified Stommel box model showing the temperature and salinity of two bodies of water as (T_1, S_1) and (T_2, S_2) . For this model, we assume $T_1 > T_2$. Ψ represents water flux due to circulation, while η represents water flux due to precipitation and evaporation.

- (a) (1 point) In which cardinal direction would you expect isopycnals to be dipping in the North Atlantic Ocean? **Explain.**

Solution: The isopycnals dip towards the south. At a given depth, the northern water column is typically denser, requiring a surface of constant density to be deeper in the south to match the northern density.

- (b) (1 point) The AMOC is at risk of collapse due to climate change. **Describe** the primary forcing mechanism by which warming is thought to induce AMOC collapse. Does the Stommel box model capture this mechanism? **Explain.**

Solution: Warming increases the rate of glacier melting. The Stommel box model does not capture this freshwater flux as it only accounts for precipitation and evaporation.

3. The Stommel model, when numerically solved, provides a bistable solution similar to that shown in Figure 3.

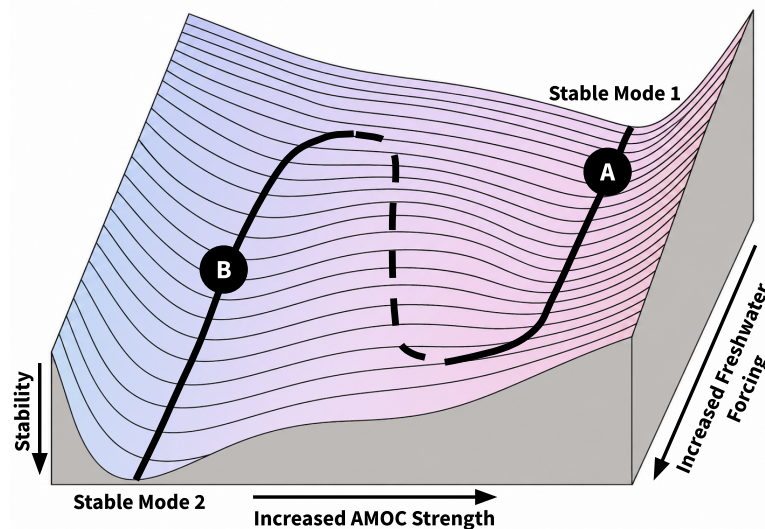


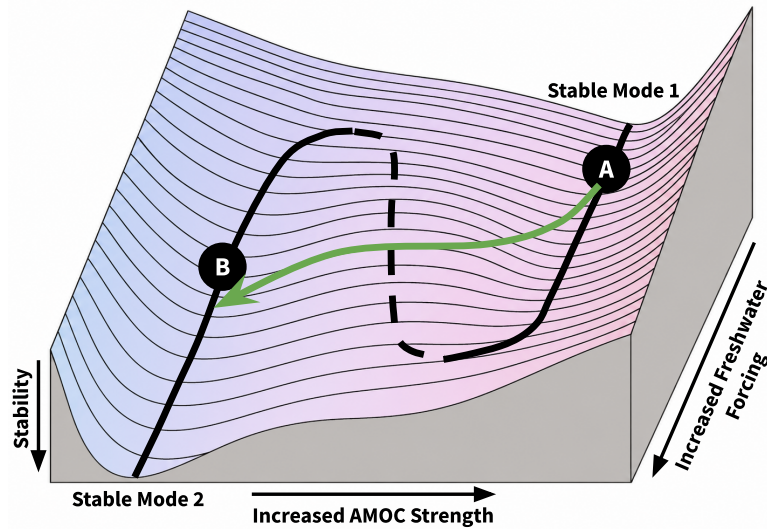
Figure 3: Stability surface plot gathered from simulations of a collapse of AMOC. Strength of AMOC at equilibrium in the stable modes is shown via the solid black lines.

- (a) (2 points) Stable Mode 1 and 2 (Figure 3, solid black) are categorized as “thermal” or “haline” based on the primary cause of the circulation. **Identify** which stable mode is the thermal one and **justify** your response.

Solution: The strong AMOC circulation today (Stable Mode 1) driven by cold water sinking at pole, which is the thermal mode. In the haline mode (Stable Mode 2), the circulation is driven by evaporation in the tropics which causes water to sink at the equator.

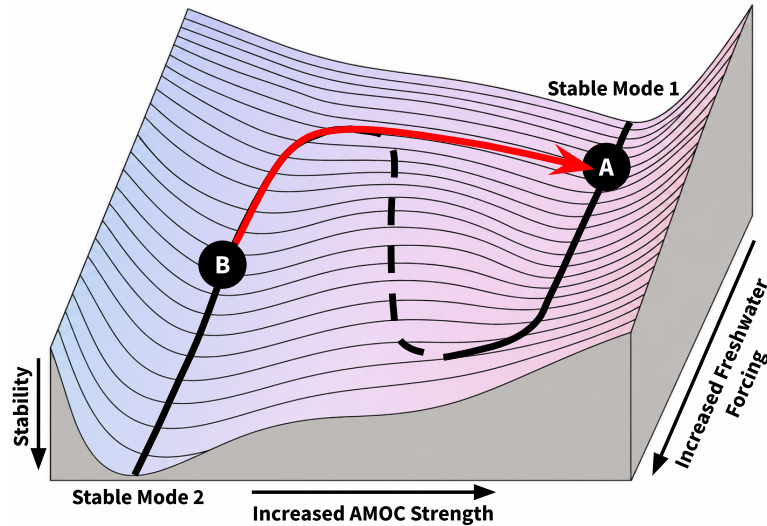
- (b) On your answer sheet, **clearly sketch** the path taken by the system in the scenarios below.
- i. (1 point) Rapid increase in freshwater forcing, causing AMOC to shift from Stable Mode 1 to Stable Mode 2, starting from point A.

Solution: Due to the rapid rate of forcing, the shift is expected to be in non-equilibrium and trend away from Stable Mode 1 before the tipping point. A strong enough non-equilibrium condition could result in crossing the unstable mode (dashed line) and equilibrating in Stable Mode 2. The green curve shows a potential non-equilibrium path for A. Note that the curve is strictly increasing in freshwater forcing.



- ii. (1 point) Slow decrease in freshwater forcing, causing AMOC to shift from Stable Mode 2 to Stable Mode 1, starting from point B.

Solution: A slow rate of forcing indicates the AMOC stays near equilibrium along Stable Mode 2 until it reaches the point where the surface is sloped such that the AMOC can only exist in Stable Mode 1. The red curve shows the expected path.



- (c) (3 points) Assume AMOC collapses via a shift from Stable Mode 1 to Stable Mode 2. In the context of the Stommel model, **describe** how this shift would impact the freshwater forcing shown in Figure 3. Does this increase or decrease the likelihood of recovery from AMOC collapse?

Solution: The collapse would decrease the transport of warm water toward poles, so the tropics become warmer and evaporation increases. In the Stommel model, greater tropical evaporation corresponds to greater polar precipitation, strengthening the freshwater forcing and decreasing the likelihood of recovery from the AMOC collapse.

4. Scientists have proposed building a physical dam across the entire Bering Strait between Russia and Alaska. A map of the proposed location is shown in Figure 4.

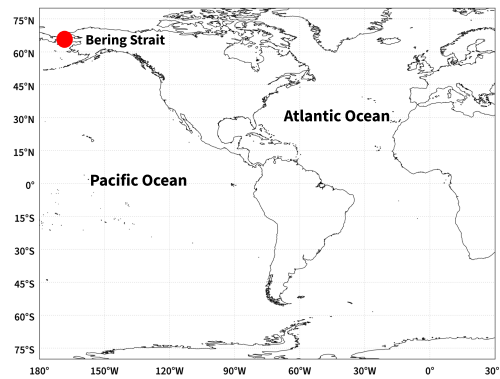


Figure 4: Global map with the Bering Strait marked with a red dot.

- (a) (2 points) Considering the global distribution of the landmasses and prevailing winds, is there a net movement of water by precipitation from the Atlantic to the Pacific or vice versa? **Explain.**

Solution: Due to high evaporation and little land separating the oceans at the tropics, exchange of water occurs most prominently here. The tropics are characterized by easterly winds which carry water from the Atlantic to the Pacific.

- (b) (2 points) The movement described in (a) is one of the primary causes of density differences between the Pacific and Atlantic. What is the expected impact of the construction of this dam on the strength of AMOC? **Explain.**

Solution: As the Pacific receives more precipitation than the Atlantic, its water is less dense. The dam would prevent mixing of the Pacific and Atlantic waters in the Arctic ocean, thus preserving the density of Atlantic surface waters to strengthen the AMOC.

5. Climate modeling can help us understand how ocean circulation will be impacted by climate change. Figure 5 depicts a model of sea surface temperature (SST) rise under an abrupt warming scenario.

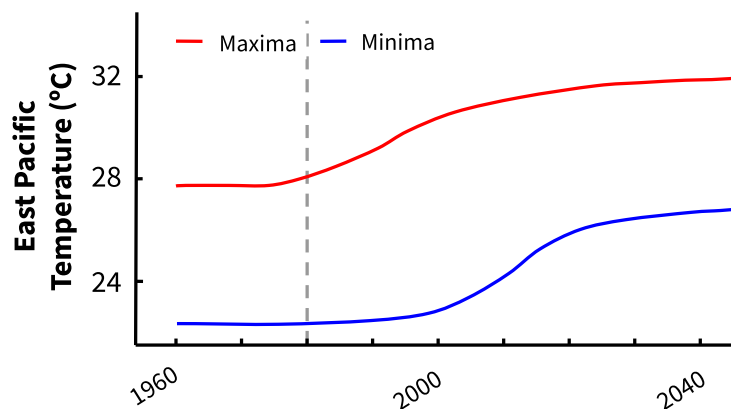


Figure 5: Results from a global climate model starting in 1960, assuming an abrupt increase in atmospheric CO_2 in 1980. Minima and maxima curves represent the minimum and maximum SST recorded each year in the equatorial East Pacific (within 5° of the equator).

- (a) (1 point) **Briefly explain** how the predicted minimum and maximum SST correlate with the El Niño Southern Oscillation (ENSO) cycle.

Solution: The minima correlate to La Niña due to increased upwelling, while the maxima correlate to El Niño due to decreased upwelling.

- (b) (2 points) **Briefly explain** why the rise in minimum SST lags behind the rise in maximum SST.

Solution: The SST minima occur when deep water upwells. Since deep water takes longer to respond to surface temperature changes, the increase in minima will lag the increase in maxima.

- (c) (1 point) Would the lag in (b) increase or decrease ENSO strength, defined as the difference between the El Niño and La Niña temperatures, during times of abrupt warming? **Briefly explain.**

Solution: The ENSO strength would increase as the maxima and minima curves would be spread further apart.

Problem 4

Question	1	2	3	Total
Points	7	7	6	20 (20%)

This question explores the properties and orbital mechanics of satellites.

1. The Hill sphere is the region within which a satellite can be gravitationally bound to a body without being pulled away by a larger body. The formula for the radius of the Hill sphere is:

$$r_H \approx r_{\text{orbital}} \sqrt[3]{\frac{m}{3M}}$$

It can be desirable to place a satellite outside Earth's Hill sphere for remote-sensing applications. One way to maintain a satellite outside this radius is with a solar sail, pictured below. In the case of a solar sail, the radiation pressure (F_{rad}) on the sail opposes the Sun's gravity, expanding the effective Hill sphere of Earth.

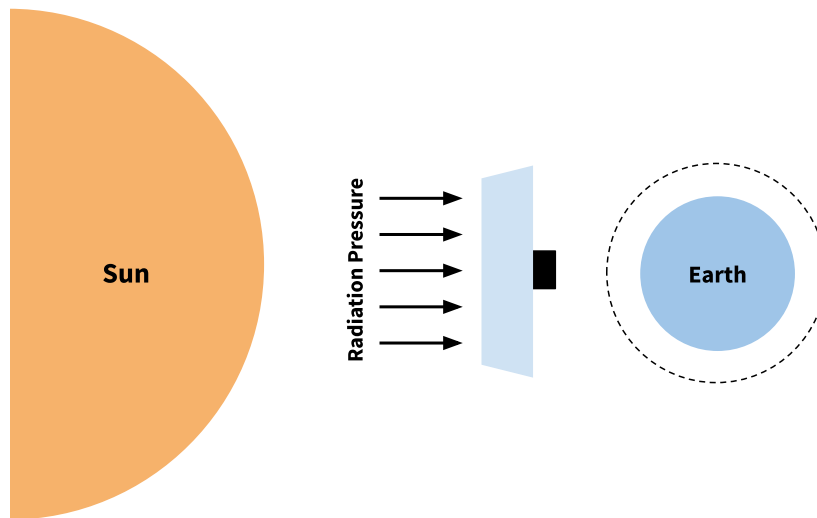


Figure 1: Proposed satellite outside Earth's Hill sphere.

- (a) (3 points) To calculate the effective Hill sphere, it can be useful to define an effective solar mass ($M_{\odot, \text{eff}}$) felt by the satellite after accounting for the effect of the solar sail. **Write** an expression for $M_{\odot, \text{eff}}$ in terms of the actual solar mass $M_{\odot}$, the force of radiation pressure F_{rad} , and the force of gravity from the Sun F_g .

Solution: The outward radiation pressure (F_{rad}) partially cancels out the inward gravitational pull of the Sun (F_g). Since $F_g \propto M_{\odot}$, the net force ($F_g - F_{\text{rad}}$) can be represented by a proportionally smaller “effective” solar mass.

$$M_{\odot, \text{eff}} = M_{\odot} \left(\frac{F_g - F_{\text{rad}}}{F_g} \right) \quad \text{or} \quad M_{\odot} \left(1 - \frac{F_{\text{rad}}}{F_g} \right)$$

- (b) (4 points) A proposed satellite of total mass 100 kg is being designed for an orbit with radius 2.0×10^9 m. Given the constants below, use the formula for F_{rad} to **calculate** the minimum area A of the solar sail required to allow the sail to maintain orbit around Earth. **Show your work.**

$$F_{\text{rad}} = \frac{2IA}{c}$$

Mass of Earth	$6.0 \times 10^{24} \text{ kg}$
Mass of the Sun	$2.0 \times 10^{30} \text{ kg}$
Orbital Radius of Earth (R_E)	$1.5 \times 10^{11} \text{ m}$
Solar Constant (I)	$1360 \frac{\text{W}}{\text{m}^2}$
Speed of Light (c)	$3.0 \times 10^8 \text{ m/s}$
Gravitational Constant (G)	$6.7 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$

Solution:

For the satellite to remain gravitationally bound to Earth at $2.0 \times 10^9 \text{ m}$, its orbit radius must equal the effective Hill-sphere radius:

$$r_H = R_E \sqrt[3]{\frac{M_E}{3M_{\odot, \text{eff}}}}$$

Solving for the effective solar mass gives:

$$\begin{aligned} M_{\odot, \text{eff}} &= \frac{M_E R_E^3}{3r_H^3} \\ &= \frac{(6.0 \times 10^{24})(1.5 \times 10^{11})^3}{3(2.0 \times 10^9)^3} \\ &= 8.4 \times 10^{29} \text{ kg} \end{aligned}$$

From part (a),

$$M_{\odot, \text{eff}} = M_{\odot} \left(1 - \frac{F_{\text{rad}}}{F_g}\right)$$

Therefore,

$$\begin{aligned} \frac{F_{\text{rad}}}{F_g} &= 1 - \frac{M_{\odot, \text{eff}}}{M_{\odot}} \\ &= 1 - \frac{8.4 \times 10^{29}}{2.0 \times 10^{30}} \\ &= 0.58 \end{aligned}$$

The Sun's gravitational force on the 100 kg satellite is:

$$\begin{aligned} F_g &= \frac{GM_{\odot}m}{R_E^2} \\ &= \frac{(6.7 \times 10^{-11})(2.0 \times 10^{30})(100)}{(1.5 \times 10^{11})^2} \\ &= 0.60 \text{ N} \end{aligned}$$

Thus, the necessary radiation-pressure force is:

$$F_{\text{rad}} = 0.58(0.60) = 0.35 \text{ N}$$

Using the given equation we solve for the sail area:

$$A = \frac{F_{\text{rad}}c}{2I} = \frac{(0.35)(3.0 \times 10^8)}{2(1360)} \approx \boxed{3.8 \times 10^4 \text{ m}^2}$$

2. Mimas and Enceladus are moons of Saturn each with a subsurface liquid water ocean sealed beneath a surface ice shell.

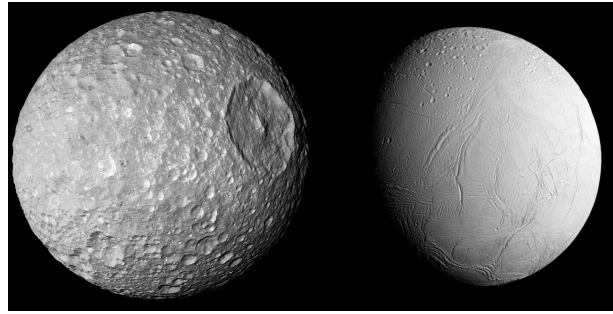


Figure 2: Images of Mimas (left) and Enceladus (right). Note that the images are not to scale; Enceladus is significantly larger than Mimas.

- (a) Tidal heating on these bodies is resulting in the melting of their ice shells.
- (2 points) What kind of stress will the melting of the ice shell put on the ductile inner layers of their ice shells? **Explain.**

Solution: Melting of surface water ice reduces the total volume of water + ice beneath the shell's surface. This creates voids that allow ice to flow inward under gravity, resulting in tensile stresses.

- (1 point) What kind of stress will the thinning of the ice shell put on the brittle surface layers of their ice shells? **Explain.**

Solution: Surface layers contract as a result of the flow from (a), creating compressive stresses.

- (1 point) Over time, these stresses can build up, resulting in tectonic activity. Based on the images in Figure 2, which of Mimas and Enceladus releases this stress via tectonic activity? **Explain.**

Solution: Enceladus, as its relative lack of craters suggests rapid surface renewal. Enceladus also possesses visible large-scale ridges from faulting and tectonic spreading.

- (b) A recent model has suggested that the stresses described in (a) may be relieved by boiling of the subsurface ocean. For this question, assume that the thickness of the ice shell is the same for both moons.

- (1 point) **Briefly describe** the mechanism that results in this boiling.

Solution: Reduced pressure from void generation lowers the boiling point.

- (2 points) **Explain** why the moon you selected in 2(a)(iii) is unable to relieve stress via this mechanism.

Solution: Enceladus' larger mass and radius results in a stronger gravitational pull on the ice shell. This increases the overburden pressure, requiring greater pressure reduction to achieve boiling. However, a large reduction in pressure often results in fractures, relieving the stress before boiling can onset.

3. Iapetus is the third most massive moon of Saturn with a radius of 735 km. Scientists observed a perfectly equatorial ridge on Iapetus's surface that wraps around approximately 75% of the moon. It is around 20 km tall and 100 km wide.

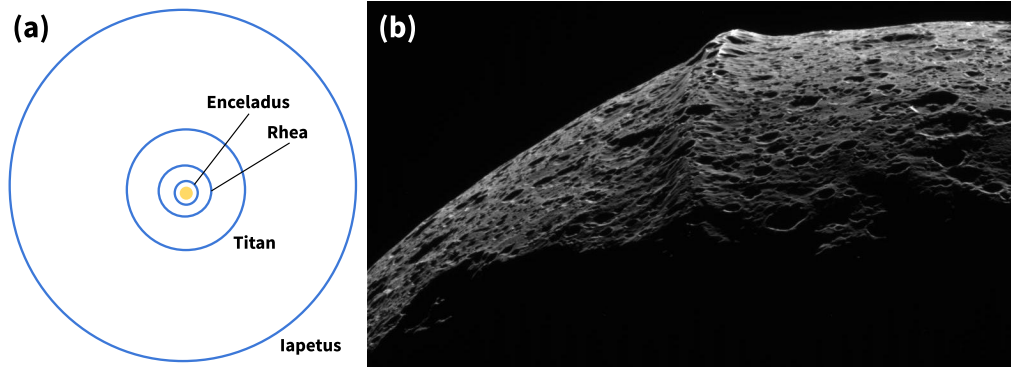


Figure 3: (a) Orbital paths of selected moons of Saturn. (b) Image of equatorial ridge of Iapetus.

- (a) (1 point) Given the following table, **calculate** the orbital period of Iapetus in days. **Show your work.**

Enceladus orbital radius	2.38×10^5 km
Enceladus orbital period	1.37 days
Iapetus orbital radius	3.56×10^6 km

Solution: By Keplers third law, $T \propto a^{\frac{3}{2}}$. Therefore

$$\frac{T_{\text{Iapetus}}}{T_{\text{Enceladus}}} = \left(\frac{a_{\text{Iapetus}}}{a_{\text{Enceladus}}} \right)^{\frac{3}{2}}$$

From the figure we can see that both Enceladus and Iapetus have roughly circular orbits, so we can plug in the given numbers to get:

$$T_{\text{Iapetus}} = 1.37 \left(\frac{3.56 \times 10^6}{2.38 \times 10^5} \right)^{\frac{3}{2}} = \boxed{79.3 \text{ days}}$$

Scientists have attempted to understand the evolution of Iapetus's equatorial ridge by modeling heat transfer throughout the moon.

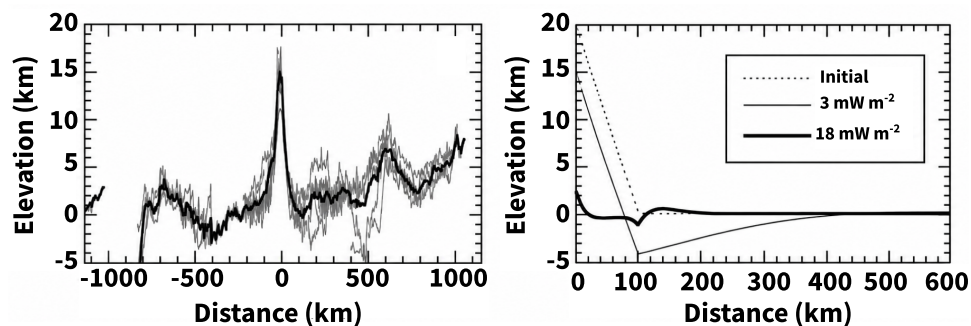


Figure 4: (a) Cross-section topography of Iapetus's equatorial ridge. (b) Simulated topographies of one side of the ridge based on different heat flow rates experienced after ridge formation.

- (b) (2 points) Based on Figures 3 and 4, did this equatorial ridge likely form shortly or long after Iapetus's formation? **Explain.**

Solution: The shown cross section is best aligned with the initial and 3mW m^{-2} curves, both of which show a prominent ridge. As a body cools down after formation, its heat flow rate reduces. Thus, the low heat flow rate given by the model implies that the ridge formed long after Iapetus's formation.

- (c) Scientists have proposed the existence of a former “subsatellite” (a satellite orbiting a moon) orbiting Iapetus's equator in order to explain the equatorial ridge.

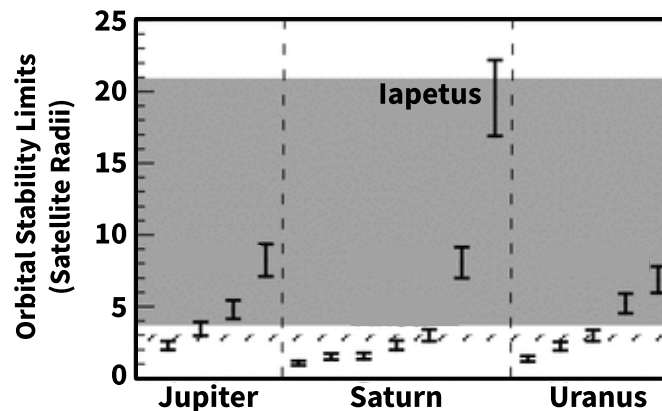


Figure 5: Orbital stability limits of notable moons in orbit of Jupiter, Saturn, and Uranus.

- i. (2 points) **Explain** why subsatellites are generally rare. What two primary parameters contribute to Iapetus being more able to sustain a subsatellite compared to other moons?

Solution: Subsateellites are rare because moons tend to have small Hill spheres. Iapetus has a large orbital radius and large mass which contribute to an increased Hill radius.

- ii. (1 point) The subsatellite is proposed to have undergone orbital decay around Iapetus. **Briefly explain** how this could have ultimately led to the formation of the equatorial ridge.

Solution: With sufficient orbital decay, the subsatellite would cross the Roche limit and break apart due to tidal forces from Iapetus. Eventually these remnants would also decay in their orbits and deposit along their orbital plane on the surface of Iapetus, forming an equatorial ridge.

Problem 5

Question	1	2	3	4	5	Total
Points	6	3	3	3	5	20 (20%)

In September 2025, NASA scientists published a discovery hinting at the possibility of life in Mars's Jezero Crater, a region that the Perseverance rover has been exploring since 2021. This question explores several aspects of that discovery and its geologic, hydrologic, and meteorologic context.

- Mars's atmospheric circulation is highly seasonal, with cells appearing and disappearing as well as changing in size between seasons. Figure 1 depicts Mars's global meridional circulation in four different seasons.

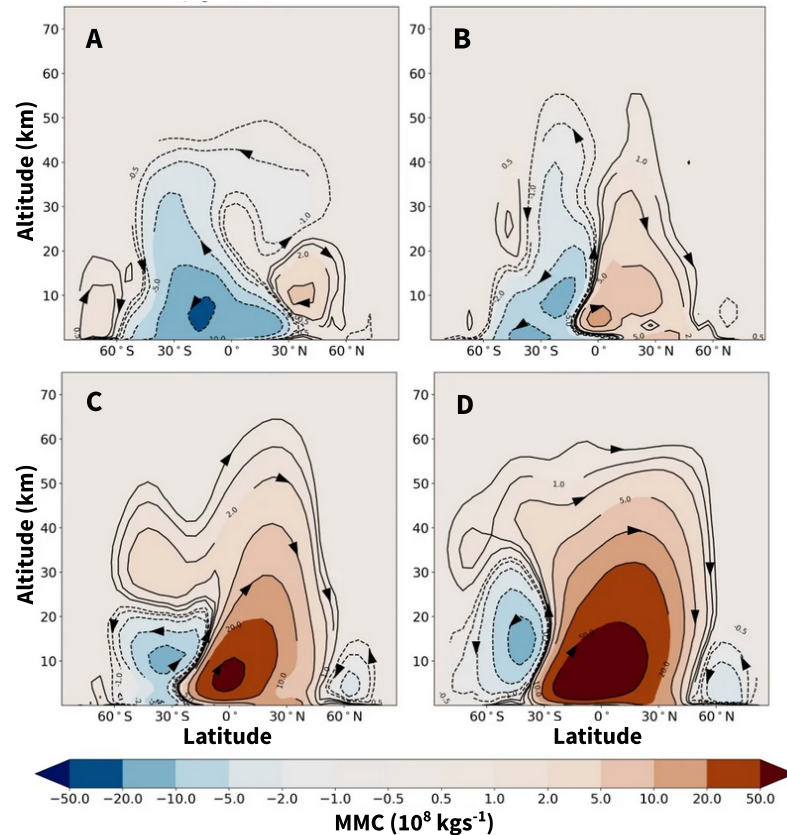


Figure 1: A model of Martian atmospheric circulation across latitudes at four points in its orbit. The shading represents mean meridional circulation (MMC); positive values represent clockwise northward flow, while negative values represent counterclockwise southward flow.

- (1 point) The most prominent circulation cells in Figure 1 are most directly analogous to which of the following types of cells on Earth?

- Hadley
- Ferrel
- Polar

Solution: The most prominent two cells in Figure 1 have opposite circulation and meet at a band of rising air centered around the equator with seasonal variations in latitude. This is analogous to Earth's Hadley cells.

- (b) (2 points) **Identify** which panels of Figure 1 represent Northern Hemisphere winter and Northern Hemisphere summer, respectively. **Explain**.

Solution: Panel A represents summer, as surface convergence takes place furthest into the Northern Hemisphere, where the sun shines with greatest intensity. Panels C and D represent the Northern Hemisphere's colder seasons, as surface convergence takes place in the Southern Hemisphere. The circulation cells in panel D are more developed than they are in panel C, so panel D represents winter.

- (c) (1 point) The orbit of Mars has a relatively high eccentricity. Is Mars closer to the Sun during Northern Hemisphere summer or winter? **Explain** how you know using Figure 1.

Solution: Atmospheric circulation is heat-driven, meaning it is strongest when the planet is closest to the Sun. Panel D (Northern Hemisphere winter) shows a much more developed circulation cell compared to Panel A.

- (d) (2 points) Jezero Crater is located at about 20°N on Mars. Ventifacts and aeolian landforms on the crater surface suggest that the dominant wind pattern has varied over time, with some landforms suggesting easterly winds and some suggesting southwesterly winds. **Identify** which of the two wind patterns is most consistent with Mars's global circulation as shown in Figure 1. **Explain** your reasoning.

Solution: The predominant wind pattern occurs in panels B, C, and D, where the circulation cell brings air from north to south at 20°N on the surface. The Coriolis effect would deflect this wind to the right, shifting it from a northerly to an easterly wind pattern. This matches the easterly pattern seen at 20°N on Earth, which is expected given both are driven by Hadley cells.

2. (3 points) Jezero Crater is on the edge of Isidis Basin, a planetary-scale depression that likely hosted an ancient ocean. The crater is thought to be a location where an ancient river (now Neretva Vallis) met the sea. The satellite map in Figure 2 shows the geology of the western Jezero Crater, with the far western edge of the crater rim seen on the left.

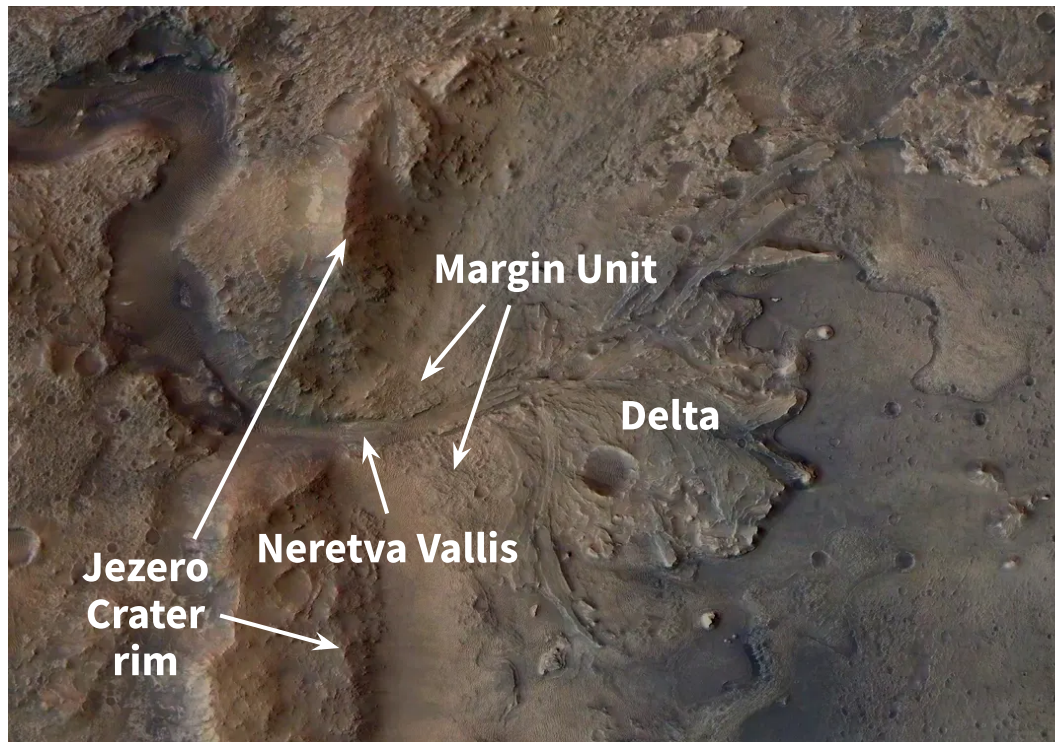


Figure 2: A map of Jezero Crater.

The highlands on both sides of the river are composed of the Margin Unit, consisting mostly of olivine and carbonate. Its olivine is assumed to be of intrusive igneous origin, while its carbonate likely originated from hydrothermal alteration of the olivine.

Using Figure 2 and the descriptions provided, **order** the following events in chronological order. **Explain** your reasoning.

- I) Formation of Jezero Crater
- II) Formation of Neretva Vallis
- III) Deposition of Curvilinear Unit delta remains
- IV) Formation of olivine-containing igneous intrusions in the Margin Unit
- V) Formation of carbonate minerals in the Margin Unit

Solution: I, IV, V, II, III. The crater is the basis for this structure and would have destroyed any remnants of the other features had it not formed first. Since the olivine formed in an intrusion, it must have formed before the region was exposed to the surface, i.e., before Neretva Vallis and the delta. Since hydrothermal alteration affects the olivine, it must happen after the intrusion's formation, but it is more likely to happen while the intrusion is buried, which would be immediately after its formation. (*Note: Students were not penalized for placing V later in the sequence.*) Finally, the intrusion must have been exposed for the river to form Neretva Vallis. Since the delta forms while the river is present, the delta forms last after Neretva Vallis is carved.

3. In 2024, Perseverance came across an outcrop in the Margin Unit called Cheyava Falls that has several unusual features. In particular, the rock contains large olivine clasts as well as so-called “leopard spots,” which contain iron sulfide minerals in their interior. Scientists believe these spots may represent reaction fronts produced by ancient anoxic bacteria feeding on iron and sulfate.

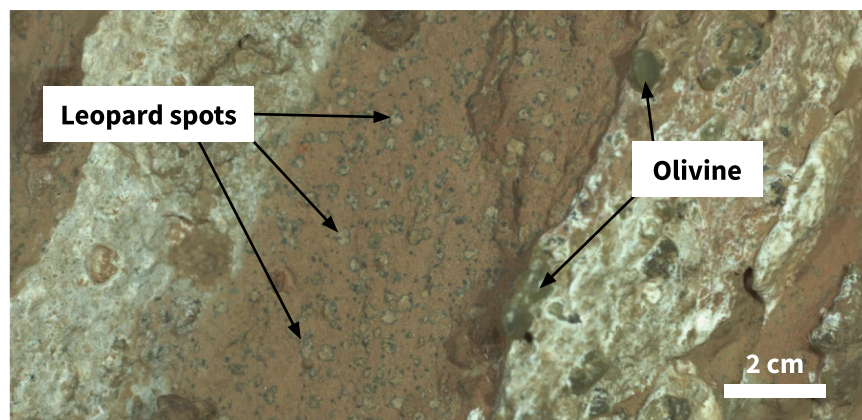


Figure 3: A photo of a small section of the Cheyava Falls rock taken by the Perseverance rover.

- (a) (1 point) **Explain** why it is rare to find olivine in sedimentary rocks on Earth but not in modern Martian sedimentary rocks.

Solution: Olivine has a low resistance to chemical weathering and tends to be weathered away quickly on Earth. Since Mars has no water to cause weathering, olivine can remain stable for much longer.

- (b) (1 point) Are the leopard spots more reduced or oxidized than the surrounding rock? **Explain** how you know.

Solution: The red color of the surrounding rock is likely due to oxidized iron. The leopard spots are much less red than the surrounding rocks, which suggest that they more reduced.

- (c) (1 point) If the iron sulfide minerals observed in the leopard spots were produced abiotically, an external source of sulfur must have been present. **Propose** a possible external source of sulfur consistent with the origin of the Margin Unit.

Solution: Sulfur is a major component of volcanic gases and fluids, which would have been present due to the previously identified igneous origin of the Margin Unit.

4. The river that created Neretva Vallis fed into Mars’s ancient ocean, which is thought to be nearly ten times as saline as Earth’s oceans today.

- (a) (1 point) **Briefly describe** the expected structure typical of an estuary fed by a freshwater river entering this saline ocean.

Solution: This would likely form a salt wedge estuary. Specifically, the estuary would have a high degree of vertical stratification, with a thin layer of freshwater extending over a larger body of dense saltwater.

- (b) (2 points) Potential Martian microorganisms are thought to have thrived in anoxic conditions. **Explain** why high salinity in the Jezero Crater lake would likely produce anoxic conditions below the lake's surface.

Solution: As stated in part (a), high salinity coupled with riverine surface freshwater input causes powerful stratification. This generally prevents whole-column mixing. As such, oxygen from the surface would not be mixed deep into the lake to replace oxygen consumed by life, causing conditions to remain anoxic.

5. It is believed that Martian tectonic activity stopped about 4 billion years ago, while water remained on the surface until about 2 billion years ago.

- (a) (2 points) During the period when water existed without plate tectonics, did geological processes produce a net increase or decrease in the carbon in Mars's atmosphere? **Explain.**

Solution: Net decrease. Water accelerates reactions that sequester carbon into geologic repositories (carbonate-silicate cycle), while plate tectonics are responsible for replenishing carbon in the atmosphere via outgassing. Without plate tectonics, carbon is steadily removed from the atmosphere without a sufficient mechanism to replace it.

- (b) The location of the solar habitable zone does not explain the loss of liquid water on Mars. In fact, the Sun has brightened over time, extending the habitable zone closer to Mars.

- i. (1 point) **Explain** how the loss of Martian surface tectonic activity eventually led to the loss of its surface water.

Solution: Consider the mechanism described in part (a). The loss of tectonic activity causes a net reduction in atmospheric CO₂, reducing the planet's greenhouse effect and causing cooling. The atmosphere eventually became so cold that no liquid water could exist on the surface.

- ii. (2 points) **Explain** how the loss of Martian internal heat eventually led to the loss of its surface water.

Solution: The loss of internal heat caused Mars to lose its magnetic field, allowing charged solar wind particles to gradually strip Mars of its atmosphere. The loss of an atmosphere reduced the planet's greenhouse effect and caused cooling. The atmosphere eventually became so cold that no liquid water could exist on the surface.

END OF SECTION II